

## UNIT- V

### POWER SYSTEM OPERATION IN COMPETITIVE ENVIRONMENT

#### 1.1. INTRODUCTION

Throughout the world, electric power utilities are currently undergoing major restructuring process and are adopting the deregulated market operation. Competition has been introduced in power systems around the world based on the promise that it will increase the efficiency of the industrial sector and reduce the cost of electrical energy of all customers. Electrical energy could not be stored in large quantities. Continuity of supply is sought as more important than the cost of the electrical energy. To meet the growing power demand, electric power industry has to adopt the deregulated structure.

For integrated operation of deregulated system, regulating agencies such as pool operator or system operator have to be formulated. In the deregulated power market, the electricity is dispatched with the help of either by a separate power exchange or the system / pool operator.

The power system deregulation is expected to offer the benefit of lower electricity price, better consumer service and improved system efficiency. However, it poses several technical challenges with respect to its conceptualization and integrated operation. Basic issues of ensuring economical, secured and stable operation of the power system, which can deliver the power at desired quality, has to be addressed carefully in a deregulated market. The complexity is more in such an arrangement.

Power systems, all over the world, have been forced to operate to their full capacities due to the environmental and/or economical constraints. This results in the need of new generation centers and transmission lines. The amount of electric power that can be transmitted between two locations through a transmission network is also limited by security constraints. Power flows should not be allowed to increase to a level where a random event could cause the network to collapse due to overloading, angular instability, voltage instability or cascaded outages. This state of the system is called as congestion of the power system. Managing congestion to minimize the restrictions of the

transmission network becomes the central activity of power system operators in recent times. The deregulation of the electric utility industry allows many independent power producers (IPP) to be connected across the transmission system. This situation also calls for effective methods to ensure the transmission system reliability, while the power is transmitted through the network.

In a deregulated environment, there are many simultaneous bilateral and multilateral transactions in addition to power pooling. Therefore, it is very much important that sellers and buyers of electricity need to find the cost allocation to their wheeling transactions. Independent System Operator (ISO), a supreme entity for the control of transmission system, also needs to know such costs in order to make correct economic and engineering decisions on upgrading the transmission facilities. So wheeling is currently a high priority problem in both regulated and deregulated power industries. Transmission Open Access (TOA) is an important step for the transition of conventional power systems to a deregulated power system. It consists of the regulatory structure, which includes transmission rights obligations, operational procedures and economic conditions of the system and enables two or more parties to use the transmission network for electricity power transfer of another party. This concept is gaining deep attention which desire to introduce competition into traditional regulated utilities without giving up their existing regulatory structure. Such a deregulated system study is carried out in the present thesis work. Before entering into the details of the work, important terms used have been explained in the following section.

#### BASIC CONCEPTS

##### *Wheeling*

Wheeling is the transmission of power from a seller to a buyer through a third party network. It may be defined as, "the use of transmission or distribution facilities of a system to transmit power of and for another entity or entities". It may also be defined as: 'Wheeling is the use of some party's

(or parties') transmission system(s) for the benefit of the other parties".

*Bilateral Wheeling Transaction* It is a bilateral exchange of power between a buying and selling entity. The exchange may be a proposed, scheduled or actual one.

#### *Multilateral Wheeling Transactions*

Multilateral transactions are an extension of bilateral transactions. In a multilateral transaction, power is injected at different buses and taken out at some other different buses simultaneously, such that the sum of all generations is equal to all loads in the transaction, excluding losses. Transmission losses may be either supplied by the generators of the transactions or by the pool utility as per predefined contract. This trade is arranged by energy brokers and involves more than two parties.

#### *Transmission Open Access (TOA)*

Because of transmission open access, entities that did not own transmission lines were granted the right to use the transmission system. The aim of TOA is to introduce competition into the traditional regulated utilities without giving up the existing regulating structure.

#### *Restructuring*

Restructuring of regulated power sector to separate the functions of power generation, transmission, distribution and electricity supply to consumers.

#### *Deregulation*

It is changing the existing monopoly franchise rule or other regulations of regulated industry, that affect how electric companies do business, and how customers may buy electric power and services.

#### *Available Transfer Capability (ATC)*

The ATC is a measure of the transfer capability remaining in the physical transmission network for further commercial activity over and above already committed uses.

#### *Total Transfer Capability (TTC)*

It is defined as the amount of electric power that can be transferred over the interconnected transmission network or particular path or interface in a reliable manner, while meeting all of a well defined pre- and postcontingency system conditions from a specified set.

#### *Transmission Reliability Margin (TRM)*

It is defined as that amount of transmission transfer capability necessary to ensure that the interconnected transmission network is secured under a reasonable range of uncertainties in the system.

#### *Capacity Benefit Margin (CBM)*

It is defined as that amount of transmission transfer capability reserved for load serving entities on the host transmission system to ensure access to generation from interconnected systems to meet generation reliability requirements.

#### *Short Run Marginal Cost (SRMC)*

Short run marginal cost of wheeling transactions for a unit megawatt in deregulated environment is calculated by taking into account the difference between bus incremental costs of the buses for producing an additional megawatt at each bus.

#### *Embedded Cost*

Embedded cost is defined as the revenue requirements needed to pay for all existing transmission facilities plus any new facilities added to the transmission network during the life of the contract for transmission service.



## Transmission System Congestion

In a competitive electricity market, congestion refers to the overloading of lines or transformers due to market settlement. The chances of congestion in the deregulated market are quite high as compared to the monopolistic market, as the customers would like to purchase electricity from the cheapest available sources. The congestion is undesirable in the system and should be alleviated for the secure operation of the system.

## DEREGULATION IN POWER INDUSTRY

The driving force behind the development of power systems is the growing demand for electrical energy in developing countries. The energy demand will be the greatest in the near future. As energy demand continues to grow, higher voltage levels are needed. In the beginning, A.C. transmission has to transfer power over long distances. In such transmission, technical problems such as voltage control and dynamic stability will arise. This involves in heavy pricing over the customer. The deregulated power system is to give opportunity to the customer to buy energy at a more favorable price.

The electric supply industry in every country for about the last one hundred years has been a natural monopoly and as a monopoly attracted regulation by government. Without exception, the industry has been operated as a vertically integrated monopoly organization that owned the generation, transmission and distribution facilities. It was also a local monopoly, in the sense that in any area one company or government agency sold electric power and services to all customers. The major difference between conventional monopolistic electricity market and the emerging deregulated market is that electricity in the former case is considered as merely energy supply sector, whereas in the latter case it is treated as a service sector and is to be marketed like any other common commodity. In a monopolistic market, the same agency is responsible for power generation, transportation, distribution as well as control, whereas in the new market structure these tasks are segregated and have to be separately paid by the transacting parties. In the conventional market, the single utility is responsible for maintaining physical flow of electricity, satisfying consumer's demands at proper voltage and frequency level, security, economy and reliability of the system. In the new deregulated electricity market, all of these tasks are treated as separate services, in addition to the primary task of the system operator and wire companies to ensure meeting the power transactions all the time. The additional services include arranging power for the loss makeup or load following, maintaining the system frequency, providing enough voltage & VAR support, arranging for start-up power, spinning reserve in the system etc.

These are called ancillary services in the deregulated environment and have to be arranged and paid separately. Some of these ancillary services can directly be arranged by the seller & buyer of electricity. In addition, the transmission of electricity itself will be treated as a separate service and has to be changed from the transacting parties and paid to the wire companies.

## Motivations for Deregulated Power Industry

Since the 1980's the electricity supply industry has been undergoing rapid and noticeable changes within the industry that is markedly stable and served the public well. A significant feature of these changes is that it allows for competition among generators and create market conditions in the industry, which are seen as necessary to reduce costs of energy production and distribution, eliminate certain inefficiencies, shed manpower and increase customer choice. This transition towards a deregulated power market is commonly referred to as electricity supply industry restructuring or deregulation. South American countries such as Argentina and Chile, were the first few to

introduce a full-scale market of electricity in the mid-eighties followed by U.K., Scandinavian countries and the USA in the 1990s, where it is now fully operational. Some of the Asian countries, including India, have already taken initial steps in this direction.

In India, a limited level of competition is already introduced at generation level by allowing participation of Independent Power Producers (IPPs). In addition, separation of three organs of power system i.e. generation, transmission and distribution has already been taken in a few states.

Shortly most of the power utilities in the country will be adopting the deregulated *structure* in some *scnsc*. Further, the regulatory bodies have been formed at central level and also at some of the states. Their primary function, at present, is to fix tariff for power sales. Many factors such as technology advances, changes in political and ideological attitudes, regulatory failures, high tariffs, managerial inadequacy, global financial drives, the rise of environmentalism, and the shortage of public resources for investment in developing countries, contributed to the worldwide bud towards deregulation.

#### Elements of Deregulated Systems

The structural components representing various segments of the deregulated electricity market are Generation companies (Gencos), Distribution companies (Discos), Scheduling Coordinator (SCs), Transmission Owners (TOs), an Independent System Operator (ISO), and a Power Exchange (PX). Gencos are responsible for operating and maintaining generating plant in the generation sector and in most of the cases are the owners of the plant. Where the transmission network was state-owned before restructuring, obviously this integrity will be retained and a distinction between owner and operator is redundant.

#### Independent System Operator (ISO)

To achieve these objectives, the ISO performs one or more of the following functions.

##### I. Power system operations function

This fundamental function includes the operation-planning function and realtime control.

##### a. Operation-planning function includes

- i. Perform power system scheduling.
- ii. Coordination with energy markets.
- iii. Perform power system dispatch.
- iv. Determine Available Transfer Capabilities (ATCs).
- v. Determine real-time ATCs.
- vi. Calculate short-term costs and prices.
- vii. Calculate hourly prices for transmission-related services.

##### b. Real-time control includes

- i. Monitor power system operation status.
- ii. Monitor system security.
- iii. Conduct physical network operations and network switching.
- iv. Deal with outages and emergencies.
- v. Coordinate real-time system operation.
- vi. Run a power pool where a can bid to buy and sell energy.
- vii. Submit the supply and load schedule to the ISO according to pre-specified protocols.

##### II. Ancillary service provision function

- i. Own certain ancillary services for satisfactory grid operation.
- ii. Purchase ancillary services transactions from market participants according to pre-specified protocols.
- iii. Provide ancillary services to transmission users.



iv. Allocate *costs* of ancillary services among all users.

UI. Transmission facilities provision function

i. Maintain the transmission network.

ii. Provide transmission facilities for all supplies and loads.

iii. Plan transmission, reactive power and FAmS expansion.

iv. Plan and commission owned ancillary services.

# ① ECONOMIC OPERATION of POWER SYSTEMS

## Introduction:

In general economy of operation is called economic dispatch problem. The main aim of economic dispatch problem is to reduce the cost of generating real power for satisfying the load and to meet transmission losses consider a thermal unit consists of boiler, turbine, alternator. The input and output characteristics of thermal unit is significant. The input to thermal unit is heat supplied or cost of fuel expressed in kcal/hr (or) Rs/hr. The output is electrical power expressed in kW (or) MW. Thermal unit has to supply a load variations of about 1% to 5% along with losses.

## System variables:

To analyze a power system network, the following are the variables which are considered.

## Control variables:

It is termed as real and reactive power generation  $P_G, Q_G$  and controls the system.

## Disturbance variables:

It is termed as real and reactive power demand  $P_D, Q_D$  and are beyond to control.



State variables:

It is termed as bus voltage magnitude ' $V$ ' and phase angle ' $\delta$ '. They are called dependent variables and being controlled by the control variables.

Optimum dispatch (or) Economic dispatch (or) optimum dispatch problem (or) Economic dispatch problem:

Economic operation is predominantly in determining the allocation of generation to each station for various loads. The first problem is unit commitment (UC) which has to be solved first and then to load scheduling.

Unit Commitment:

It is not economical to run all the units at all the time for supplying a particular load. The unit of which has to be operated is the problem of unit commitment (UC) and it is significant for thermal unit.

The UC problem can be solved by turning on most efficient plan followed by less efficient plans.

The combination of units for supplying a particular load is to try all the possible combinations by using coordination equations.

## Load scheduling:

Scheduling is the process of allocation of generation from different generating units. It is one of the cost effective mode of technique so that allocation is done to minimize the cost.

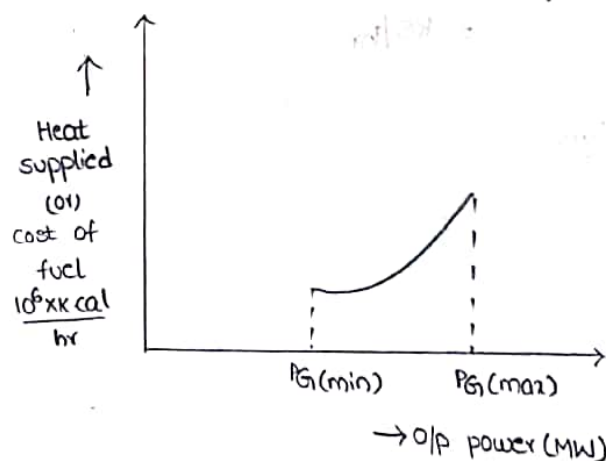
Let  $P_D \rightarrow$  Total load demand

$n \rightarrow$  no. of generating units

$P_{G1}, P_{G2}, P_{G3}, \dots, P_{Gn} \rightarrow$  Individual generating units

In optimization the total load demand has to be shared on all the individual generating units.

## Input and output characteristics:



The idealised form of input and output characteristics of steam turbine is shown in fig. It gives the relation between energy input to the turbine i.e., Heat supplied or cost of fuel expressed in  $\frac{10^6 \times \text{kcal}}{\text{hr}}$  or Rs/hr and output is electrical power expressed in MW.



Input-Output curve is not smooth curve and it has limits of power output ( $P_{Gmin}$  and  $P_{Gmax}$ ) which depends up on the following factors.

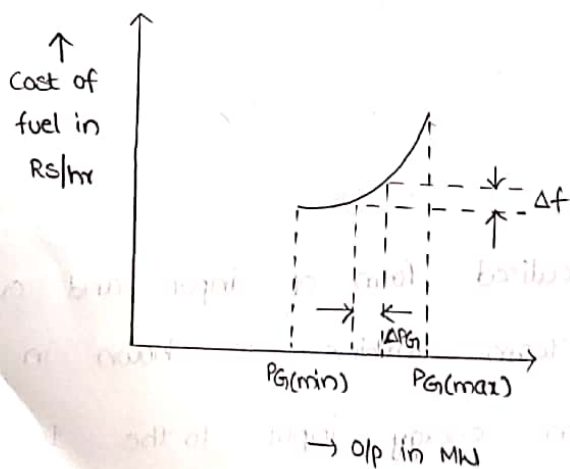
- Steam cycle used.
- Thermal characteristics of material.
- Temperature.

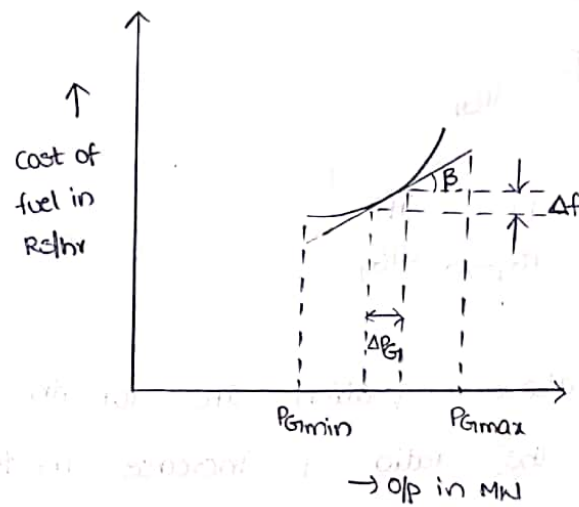
Cost curve:

It is obtained from input and output curve the cost of fuel is multiplied by fuel input.

$$\text{Fuel input} \times \text{cost of fuel} = \frac{10^6 \text{ kcal}}{\text{hr}} \times \frac{\text{Rs}}{10^6 \text{ kcal}} \\ = \text{Rs/hr}$$

Incremental fuel cost curve:





- Incremental fuel cost can be obtained from the input and output curves.
- IFC is defined as the ratio of small change in input to the small change in output.

$$IFC = \frac{\Delta \text{input}}{\Delta \text{output}}$$

$\Delta$  is progressively very small hence we consider

$$IFC = \frac{d(\text{ilp})}{d(\text{olp})} = \frac{df}{dP_{gi}}$$

IFC = slope of ilp and o/p curve i.e. the change in cost of fuel to power output at different points on the ilp and o/p curve.

$$IFC = (IC)_i$$

= slope of ilp and o/p curve



From figure (b)

$$\tan \beta = \frac{\Delta f}{\Delta P_{Gi}}$$

$$\text{IFC} = (\text{IC})_i = \lim_{P_{Gi} \rightarrow 0} \frac{df_i}{dP_{Gi}}$$

From the above equation IFC for  $i$ th thermal unit is defined as the ratio of increase in fuel input to the power output with limits  $P_{Gi} \rightarrow 0$

$$I_i = \frac{df_i}{dP_{Gi}}$$

$$I_i = \frac{dc_i}{dP_{Gi}}$$

$$\frac{df_i}{dP_{Gi}} = \frac{dc_i}{dP_{Gi}}$$

$$(\because df_i = dc_i)$$

Mathematically cost curve expression

$$C_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i$$

Differentiate above equation with respect to  $P_{Gi}$

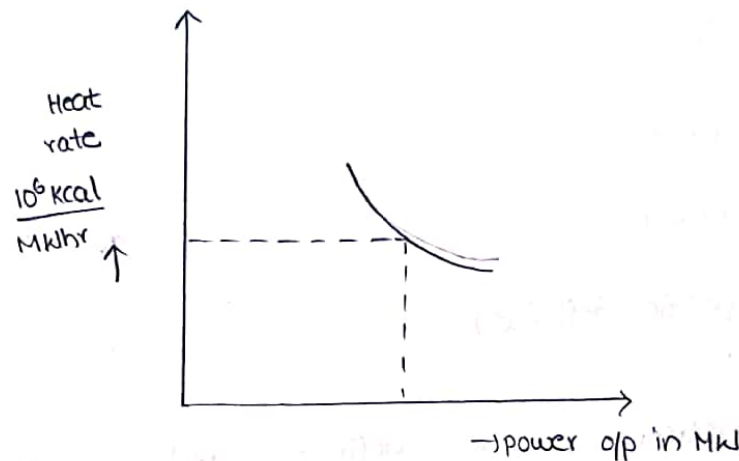
$$\frac{dc_i}{dP_{Gi}} = \frac{1}{2} a_i 2P_{Gi} + b_i$$

$$\frac{dc_i}{dP_{Gi}} = a_i P_{Gi} + b_i$$

$\frac{dc_i}{dP_{Gi}}$  = Incremental heat rate curve

IFC is very significant in economic loading of generating units.

Heat rate curve:



It gives the ratio between heat rate in

$\frac{10^6 \text{ kcal}}{\text{MWhr}}$  and power output.

$$\Rightarrow \frac{10^6 \text{ kcal}}{\text{MWhr}} \times \text{MW}$$

$$= \frac{10^6 \text{ kcal}}{\text{hr}}$$

Incremental efficiency:

It is the reciprocal of incremental heat rate.

$$\text{Incremental efficiency} = \frac{dP_{Gi}}{dF_i}$$

Incremental production Cost:

Incremental production cost (IPC) is the sum of IFC and cost of labour, suppliers and maintainance. There is no any mathematical method for calculating cost of labour, suppliers and maintainance. Hence  $IFC = IPC$



## System constraints:

i) Equality constraints.

ii) Inequality constraints

a) Soft type (flexible)

b) Hard type (specific, definite)

- Hard type constraints are definite and specific like tap changing of an load tap changing transformer.
- Soft type constraints are flexible and determines nodal voltages, phase angle between nodal voltages. They can be effectively used by using penalty factor method.

## Equality constraints:

The basic load flow equations for equality constraints is

$$P_p = \sum_{q=1}^n \left[ e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq}) \right]$$

$$Q_p = \sum_{q=1}^n \left[ f_p(e_q G_{pq} + f_q B_{pq}) - e_p(f_q G_{pq} - e_q B_{pq}) \right]$$

Where,  $e_p, f_p$  are real and imaginary parts of voltages at  $p$ th node.

$e_q, f_q$  are real and imaginary parts of voltages at  $q$ th node.

$G_{pq}, B_{pq}$  are nodal conductance and susceptance between  $p$  and  $q$  nodes.

### Inequality constraints:

- i) Generator constraints
- ii) voltage constraints.
- iii) Running spare capacity constraints
- iv) Transformer tap setting constraints.
- v) Transmission line constraints.
- vi) Network security constraints

#### i) Generator constraints:

The kVA rating of the generator is

$$S^2 = P_p^2 + Q_p^2$$

$$S = \sqrt{P_p^2 + Q_p^2}$$

The kVA rating is always less than prespecified value  $C_p$  due to temperature variations.

Maximum active power is limited by thermal consideration and minimum active power can be limited by flame instability of the boiler.

The active power is rated by

$$P_{min} \leq P_p \leq P_{max}$$

The maximum reactive power is limited by rotor and the minimum reactive power is limited by stability of the machine.

Reactive power can't be outside the range and stated by

$$Q_{pmin} \leq Q_p \leq Q_{pmax}$$

ii) voltage constraints:

It is essential that voltage magnitude and phase angle must be within certain limits.

If the voltage magnitude varies then

- The equipment may not work properly connected in that system.
- The voltage regulating devices becomes uneconomical.

$|V_p|$  = voltage magnitude

It lies between  $|V_p|_{min} \leq |V_p| \leq |V_p|_{max}$

$\delta_p$  = phase angle

It lies between  $\delta_{pmin} \leq \delta_p \leq \delta_{pmax}$

Maximum value of phase angle is  $30^\circ - 45^\circ$

Minimum values assures that proper utilisation of transmission line capacity.



### (iii) Running spare capacity constraints:

These constraints can be able to withstand

- Unexpected load on the system.
- Forced outages of one or more alternators generation has to meet load as well as losses.

Total generation is stated by

$$G_1 \geq P_p + P_{s0}$$

Where  $P_p$  is active power

$P_{s0}$  indicates pre-specified power

- There must be a minimum spare capacity must be available.

### iv) Transformer tap setting constraints:

- If auto-transformer is used then minimum tap setting is zero and maximum tap setting is one stated by  $0 \leq t \leq 1$
- For two winding transformer tapplings are provided on the secondary and stated by  $0 \leq t \leq n$

Where 'n' is the transformation ratio for phase shifting transformer, the phase shift is stated as

$$\theta_{pmin} \leq \theta_p \leq \theta_{pmax}$$

#### v) Transmission Line constraints:

The active and reactive power is limited by thermal stability which is stated by

$$C_p \leq C_{pmax}$$

Where  $C_p$  - Prespecified power

$C_{pmax}$  - Maximum loading constraints

#### vi) Network security constraints:

- If the system is operating satisfactorily then if there is any outage some of the constraints may be violated.
- The complexity of the constraints is increased when a large interconnected system is used.
- The nature of the constraints is same as voltage and transmission line.

Economic dispatch with neglecting losses (or) Optimal dispatch

without losses:

Let  $F_t$  = total Fuel input

$F_n$  = Fuel i/p of 'n' generating units

$P_D$  = Total load demand

$P_n$  = generation of nth unit

Economic dispatch is defined by

$$\text{Min } F_t = \sum_{n=1}^n F_n$$

$$P_D = \sum_{n=1}^n P_n$$

By using Lagrangian multiplier

Auxiliary function

$$F = F_t + \lambda \left( P_D - \sum_{n=1}^n P_n \right)$$

Differentiating the above eqn with respect to ' $P_n$ ' and equivalent it to zero.

$$\frac{dF}{dP_n} = \frac{dF_t}{dP_n} + \lambda(0-1) = 0$$

$$\frac{dF}{dP_n} = \frac{dF_t}{dP_n} - \lambda = 0$$

$$\frac{dF_t}{dP_n} = \lambda$$



$$F_t = F_1 + F_2 + \dots + F_n$$

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_3}{dP_3} = \dots = \frac{dF_n}{dP_n} = \lambda$$

$$\boxed{\frac{dF}{dP_n} = \frac{dF_t}{dP_n} = \lambda}$$

Incremental fuel cost is expressed in Rs/MWhr.

Incremental fuel cost over a limited range.

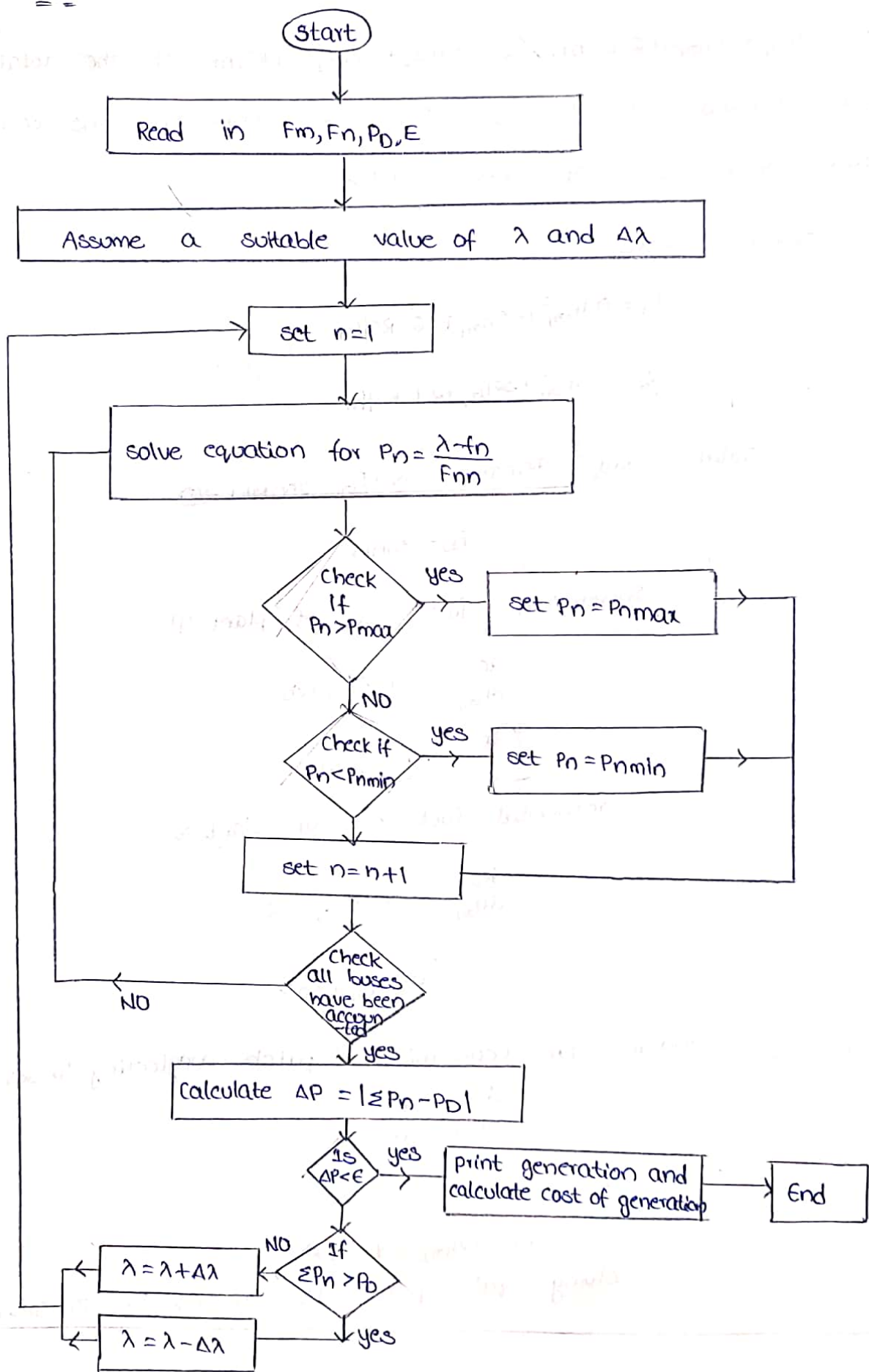
$$\frac{dF_n}{dP_n} = F_{nn}P_n + f_n$$

Where  $F_{nn}$  = slope of incremental fuel cost

$f_n$  = Intercept of incremental fuel cost

By solving simultaneous equations, we obtain economic operating schedule. A good technique for solving linear equations along with constraints the following iterative method is used.

Flow chart:



• The fuel cost of two units is given by

$C_1 = 0.1P_{G1}^2 + 25P_{G1} + 1.6$  Rs/hr,  $C_2 = 0.1P_{G2}^2 + 32P_{G2} + 2.1$  Rs/hr. If the total load demand on the generators is 250 MW. Find the economic load distribution of two units.

Sol: Given costs

$$C_1 = 0.1P_{G1}^2 + 25P_{G1} + 1.6 \text{ Rs/hr}$$

$$C_2 = 0.1P_{G2}^2 + 32P_{G2} + 2.1 \text{ Rs/hr}$$

Total load demand  $P_{G1} + P_{G2} = 250 \text{ MW} \rightarrow \textcircled{1}$

$$P_D = 250 \text{ MW}$$

Incremental fuel cost of plant '1'

$$\begin{aligned} \frac{dC_1}{dP_{G1}} &= 0.1 \times 2P_{G1} + 25 \\ &= 0.2P_{G1} + 25 \end{aligned}$$

Incremental fuel cost of 'plant 2'

$$\begin{aligned} \frac{dC_2}{dP_{G2}} &= 0.1 \times 2P_{G2} + 32 \\ &= 0.2P_{G2} + 32 \end{aligned}$$

Condition for economic dispatch neglecting losses

$$\frac{dC_1}{dP_{G1}} = \frac{dC_2}{dP_{G2}}$$

$$0.2P_{G1} + 25 = 0.2P_{G2} + 32$$

$$0.2P_{G1} - 0.2P_{G2} = 7 \rightarrow \textcircled{2}$$

solving eq ① & eq ②  $\Rightarrow P_{G1} = 142.5 \text{ MW}, P_{G2} = 107.5 \text{ MW}$



- The fuel cost of 2 units are given by  $C_1 = 0.2P_{G1}^2 + 25P_{G1} + 1$  Rs/hr,  
 $C_2 = 0.2P_{G2}^2 + 35P_{G2} + 1.5$  Rs/hr. If the total load demand on the  
 generators is 200MW. Find the economic load scheduling of  
 two units.

Sol:

Given,

$$C_1 = 0.2P_{G1}^2 + 25P_{G1} + 1 \text{ Rs/hr}$$

$$C_2 = 0.2P_{G2}^2 + 35P_{G2} + 1.5 \text{ Rs/hr}$$

$$\text{Total load demand, } P_{G1} + P_{G2} = 200 \text{ MW} \rightarrow (1)$$

Incremental cost of plant '1'

$$\frac{dC_1}{dP_{G1}} = 0.2 \times 2P_{G1} + 25$$

$$= 0.4P_{G1} + 25$$

Incremental cost of plant '2'

$$\frac{dC_2}{dP_{G2}} = 0.4P_{G2} + 35$$

condition for economic dispatch

$$\frac{dC_1}{dP_{G1}} = \frac{dC_2}{dP_{G2}}$$

$$0.4P_{G1} + 25 = 0.4P_{G2} + 35$$

$$0.4P_{G1} - 0.4P_{G2} = 10 \rightarrow (2)$$

Solving eq (1) & eq (2)

$$P_{G1} = 112.5 \text{ MW}$$

$$P_{G2} = 87.5 \text{ MW}$$

A plant has two generators neither is to be operated below 20 or above 125MW. Incremental cost of 2 units are

$$\frac{dc_1}{dP_{G1}} = 0.15P_{G1} + 20 \text{ RS/MWhr}, \quad \frac{dc_2}{dP_{G2}} = 0.225P_{G2} + 17.5 \text{ RS/MWhr. For economic}$$

dispatch. Find the plant cost of received power in RS/MWhx2)

when  $P_{G1} + P_{G2}$  equal to a) 40MW b) 100MW c) 225MW.

Sol:

Given,

$$P_{G1} > 20 \text{ MW}$$

$$P_{G2} < 125 \text{ MW}$$

condition for economic dispatch

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$$

$$0.12P_{G1} + 20 = 0.225P_{G2} + 17.5$$

$$0.15P_{G1} - 0.225P_{G2} = -2.5 \rightarrow (1)$$

$$a) \quad P_{G1} + P_{G2} = 40 \text{ MW} \rightarrow (2)$$

solve eq (1) & eq (2)

$$P_{G1} = 17.3 \text{ MW}$$

$$P_{G2} = 22.6 \text{ MW}$$

since  $P_{G2} < 125 \text{ MW}$

$$\text{we use } \frac{dc_2}{dP_{G2}} = \lambda$$

$$\text{Cost of received power } \lambda = 0.225P_{G2} + 17.5$$

$$= 0.225 \times 22.6 + 17.5$$

$$\lambda = 22.5 \text{ RS/MWhr}$$

b)  $P_{G1} + P_{G2} = 100 \text{ MW} \rightarrow \textcircled{3}$

solve eq ① & eq ③

$$P_{G1} = 53.3 \text{ MW}$$

$$P_{G2} = 46.6 \text{ MW}$$

$$P_{G1} > 20 \text{ MW}, P_{G2} < 125 \text{ MW}$$

$$\frac{dc_2}{dP_{G2}} = \lambda$$

$$\lambda = 0.225 P_{G2} + 17.5$$

$$= 0.225(46.6) + 17.5$$

$$\lambda = 27.985 \text{ RS/MWhr}$$

c)  $P_{G1} + P_{G2} = 225 \text{ MW} \rightarrow \textcircled{4}$

solve eq ① & eq ④

$$P_{G1} = 128.33 \text{ MW}$$

$$P_{G2} = 96.6 \text{ MW}$$

$$P_{G1} > 20 \text{ MW}$$

$$P_{G2} < 125 \text{ MW}$$

$$\frac{dc_2}{dP_{G2}} = \lambda$$

$$\lambda = 0.225 P_{G2} + 17.5$$

$$= 0.225(96.6) + 17.5$$

$$\lambda = 39.235 \text{ RS/MWhr}$$



- Three plants of total capacity 500MW are scheduled for operation to supply a total system load of 310MW. Evaluate the optimum load scheduling if the plants have following cost characteristics and limitations.

i)  $C_1 = 0.06P_{G1}^2 + 30P_{G1} + 10$  RS/MWhr,  $30 \leq P_{G1} \leq 150$

ii)  $C_2 = 0.10P_{G2}^2 + 40P_{G2} + 15$  RS/MWhr,  $20 \leq P_{G2} \leq 100$

iii)  $C_3 = 0.075P_{G3}^2 + 10P_{G3} + 20$  RS/MWhr,  $50 \leq P_{G3} \leq 250$

Sol:

Given,

$$C_1 = 0.06P_{G1}^2 + 30P_{G1} + 10 \text{ RS/MWhr}$$

Incremental cost of plant '1'

$$\frac{dC_1}{dP_{G1}} = 0.06 \times 2P_{G1} + 30$$

$$= 0.12P_{G1} + 30 \rightarrow (1)$$

$$C_2 = 0.10P_{G2}^2 + 40P_{G2} + 15 \text{ RS/MWhr}$$

Incremental cost of plant - '2'

$$\frac{dC_2}{dP_{G2}} = 0.10 \times 2P_{G2} + 40$$

$$= 0.2P_{G2} + 40 \rightarrow (2)$$

$$C_3 = 0.075P_{G3}^2 + 10P_{G3} + 20 \text{ RS/MWhr}$$

Incremental cost of plant - '3'

$$\frac{dC_3}{dP_{G3}} = 0.15P_{G3} + 10 \rightarrow (3)$$

$$P_D = P_{G1} + P_{G2} + P_{G3} = 310 \text{ MW}$$

$$P_{G1} = 310 - P_{G2} - P_{G3} \rightarrow (4)$$

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}} = \frac{dc_3}{dP_{G3}}$$

$$\Rightarrow \frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$$

$$0.12 P_{G1} + 30 = 0.2 P_{G2} + 40$$

$$0.12 P_{G1} - 0.2 P_{G2} = 10 \rightarrow (5)$$

Substitute eq(4) in eq (5)

$$0.12(310 - P_{G2} - P_{G3}) - 0.2 P_{G2} = 10$$

$$37.2 P_{G2} - 0.12 P_{G3} = -27.2 \rightarrow (6)$$

$$\frac{dc_2}{dP_{G2}} = \frac{dc_3}{dP_{G3}}$$

$$0.2 P_{G2} + 40 = 0.15 P_{G3} + 10$$

$$0.2 P_{G2} - 0.15 P_{G3} = -30 \rightarrow (7)$$

solving eq(6) & eq(7)

$$P_{G2} = 6.66 \text{ MW}$$

$$P_{G3} = 208.8 \text{ MW}$$

$$P_{G1} = 310 - 6.66 - 208.8 = 94.4 \text{ MW}$$

The obtained values of  $P_{G1}$  &  $P_{G3}$  exists b/w the limits but  $P_{G2} < 20 \text{ MW}$ . So, we consider  $P_{G2} = 0$

$$P_{G1} + P_{G2} + P_{G3} = 310 \text{ MW}$$

$$\Rightarrow P_{G1} + P_{G3} = 310 - 20$$

$$\Rightarrow P_{G1} + P_{G3} = 290 \rightarrow \textcircled{8}$$

Solve eq  $\textcircled{5}$ ,  $\textcircled{7}$ ,  $\textcircled{8}$  then

$$P_{G1} = 87.03 \text{ MW}$$

$$P_{G2} = 2.22 \text{ MW}$$

$$P_{G3} = 202.96 \text{ MW}$$

- Three power plants of total capacity 500MW are scheduled for operation to supply the total load of 350MW. Find optimal load scheduling if the plants have following IFC and generator constraints.

$$\text{i) } \frac{dc_1}{dP_{G1}} = 0.25P_{G1} + 40, \quad 30 \leq P_{G1} \leq 150$$

$$\text{ii) } \frac{dc_2}{dP_{G2}} = 0.20P_{G2} + 50, \quad 40 \leq P_{G2} \leq 125$$

$$\text{iii) } \frac{dc_3}{dP_{G3}} = 20 + 0.20P_{G3}, \quad 50 \leq P_{G3} \leq 225$$



Sol:

Given,

$$\frac{dc_1}{dP_{G1}} = 0.25P_{G1} + 40 \rightarrow (1)$$

$$\frac{dc_2}{dP_{G2}} = 0.30P_{G2} + 50 \rightarrow (2)$$

$$\frac{dc_3}{dP_{G3}} = 20 + 0.20P_{G3} \rightarrow (3)$$

$$P_D = P_{G1} + P_{G2} + P_{G3} = 350 \text{ MW}$$

$$P_{G1} = 350 - P_{G2} - P_{G3} \rightarrow (4)$$

For economic load scheduling,

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}} = \frac{dc_3}{dP_{G3}}$$

$$\Rightarrow \frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$$

$$0.25P_{G1} + 40 = 0.30P_{G2} + 50$$

$$0.25P_{G1} - 0.30P_{G2} = 10 \rightarrow (5)$$

Substitute eq(4) in eq(5)

$$0.25(350 - P_{G2} - P_{G3}) - 0.30P_{G2} = 10$$

$$87.5 - 0.25P_{G2} - 0.25P_{G3} - 0.30P_{G2} = 10$$

$$-0.25P_{G3} - 0.55P_{G2} = -77.5 \rightarrow (6)$$

$$\frac{dc_2}{dP_{G2}} = \frac{dc_3}{dP_{G3}}$$

$$0.30P_{G2} + 50 = 0.2P_{G3} + 20$$

$$0.3P_{G2} - 0.2P_{G3} = -30 \rightarrow \text{⑦}$$

Solving eq ⑥ & eq ⑦

$$P_{G2} = 43.24 \text{ MW}$$

$$P_{G3} = 214.86 \text{ MW}$$

$$P_{G1} = 350 - 43.24 - 214.86$$

$$= 91.9 \text{ MW}$$

- The incremental cost characteristics of two thermal plants are  $\frac{dc_1}{dP_{G1}} = 0.2P_{G1} + 60$  Rs/MWhr,  $\frac{dc_2}{dP_{G2}} = 0.3P_{G2} + 40$  Rs/MWhr. Calculate the sharing of load of 200MW for most economic operations. If the plants are rated 150, 250MW respectively. What will be the sharing in cost in rupees/hr in comparison to the loading in the same proportion to the rating.

Sol:

Given,

IFC of two plants

$$\frac{dc_1}{dP_{G1}} = 0.2P_{G1} + 60 \text{ Rs/MWhr}$$

$$\frac{dc_2}{dP_{G2}} = 0.3P_{G2} + 40 \text{ Rs/MWhr}$$

$$P_D = P_{G1} + P_{G2} = 200 \text{ MW} \rightarrow \textcircled{1}$$

For economic load scheduling

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$$

$$0.2P_{G1} + 60 = 0.3P_{G2} + 40$$

$$0.2P_{G1} - 0.3P_{G2} = -20 \rightarrow \textcircled{2}$$

solving eq ① & eq ②

$$P_{G1} = 80 \text{ MW}$$

$$P_{G2} = 120 \text{ MW}$$



Actual rating of Generator

$$P_{G1} = 150 \text{ MW}, P_{G2} = 250 \text{ MW}$$

Cost of plant '1'

$$C_1 = \int_{80}^{150} (0.2 P_{G1} + 60) dP_{G1}$$

$$= 0.2 \left[ \frac{P_{G1}^2}{2} \right]_{80}^{150} + 60 [P_{G1}]_{80}^{150}$$

$$= 0.2 \left[ \frac{(150)^2}{2} - \frac{(80)^2}{2} \right] + 60(150 - 80)$$

$$C_1 = 5810 \text{ Rs/hr}$$

Cost of plant '2'

$$C_2 = \int_{120}^{250} (0.3 P_{G2} + 40) dP_{G2}$$

$$= 0.3 \left[ \frac{P_{G2}^2}{2} \right]_{120}^{250} + 40 [P_{G2}]_{120}^{250}$$

$$= 0.3 \left[ \frac{(250)^2}{2} - \frac{(120)^2}{2} \right] + 40(250 - 120)$$

$$C_2 = 12415 \text{ Rs/hr}$$

$$\text{Saving in Cost} = 12415 - 5810$$

$$= 6605 \text{ Rs/hr}$$

The incremental cost of two units are  $\frac{dc_1}{dP_{G1}} = 0.15P_{G1} + 25$ ,  
 $\frac{dc_2}{dP_{G2}} = 0.20P_{G2} + 28$ . Assume continuous running with a total load of 150MW. Calculate the saving in cost obtained most economical division of load between the units as compared with loading each equally. The maximum and minimum operational loadings are same for each unit and are 125, 20MW.

Sol:

Given,

$$\frac{dc_1}{dP_{G1}} = 0.15P_{G1} + 25$$

$$\frac{dc_2}{dP_{G2}} = 0.20P_{G2} + 28$$

$$P_D = P_{G1} + P_{G2} = 150 \text{ MW} \rightarrow (1)$$

For economic load scheduling

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$$

$$0.15P_{G1} + 25 = 0.20P_{G2} + 28$$

$$0.15P_{G1} - 0.20P_{G2} = 3 \rightarrow (2)$$

Solve eq (1) & eq (2)

$$P_{G1} = 94.28 \text{ MW}, P_{G2} = 55.71 \text{ MW}$$

equal load for both units

$$P_{G1} = P_{G2} = 75 \text{ MW}$$

cost of plant-1:

$$C_1 = \int_{94.3}^{75} (0.15 P_{G1} + 25) dP_{G1}$$

$$= - \left[ 0.15 \left( \frac{P_{G1}^2}{2} \right)_{94.3}^{75} + 25 (P_{G1})_{94.3}^{75} \right]$$

$$= - \left[ 0.15 \left( \frac{75^2 - 94.3^2}{2} \right) + 25 (75 - 94.3) \right]$$

$$C_1 = 727.56 \text{ RS/hr}$$

cost of plant-2:

$$C_2 = \int_{55.71}^{75} (0.2 P_{G2} + 28) dP_{G2}$$

$$= \left[ 0.2 \left( \frac{P_{G2}^2}{2} \right)_{55.71}^{75} + 28 (P_{G2})_{55.71}^{75} \right]$$

$$C_2 = 792.26 \text{ RS/hr}$$

saving in cost =  $C_2 - C_1$

$$= 792.26 - 727.56$$

$$= 65.87 \text{ RS/hr}$$



- The incremental fuel cost in RS/MWhr for a plant consisting of two units are  $\frac{dc_1}{dP_{G1}} = 0.2P_{G1} + 40$ ,  $\frac{dc_2}{dP_{G2}} = 0.25P_{G2} + 30$ . Calculate the extra cost increased in RS/hr. If a load of 220MW is scheduled as  $P_{G1} = P_{G2} = 110\text{MW}$ .

Sol:

Given,

$$\frac{dc_1}{dP_{G1}} = 0.2P_{G1} + 40 \text{ RS/MWhr}$$

$$\frac{dc_2}{dP_{G2}} = 0.25P_{G2} + 30 \text{ RS/MWhr}$$

$$P_D = P_{G1} + P_{G2} = 220\text{MW} \rightarrow (1)$$

For economic load scheduling

$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}}$$

$$0.2P_{G1} + 40 = 0.25P_{G2} + 30$$

$$0.2P_{G1} - 0.25P_{G2} = -10 \rightarrow (2)$$

solving eq (1) & eq (2)

$$P_{G1} = 100\text{MW}$$

$$P_{G2} = 120\text{MW}$$

cost of plant-1,

$$C_1 = \int_{100}^{110} (0.2P_{G1} + 40) dP_{G1}$$

$$= \left[ 0.2 \frac{(P_{G1})^2}{2} + 40P_{G1} \right]_{100}^{110}$$

$$C_1 = 0.2 \left( \frac{(110)^2 - (100)^2}{2} \right) + 40(110 - 100)$$

$$= 610 \text{ RS/hr}$$

cost of plant 2,

$$C_2 = - \int_{120}^{110} (0.25 P_{G2} + 30) dP_{G2}$$

$$= - \left[ 0.25 \left( \frac{P_{G2}^2}{2} \right) + 30 P_{G2} \right]_{120}^{110}$$

$$= - \left[ 0.25 \left[ \frac{(110)^2}{2} - \frac{(120)^2}{2} \right] + 30(110 - 120) \right]$$

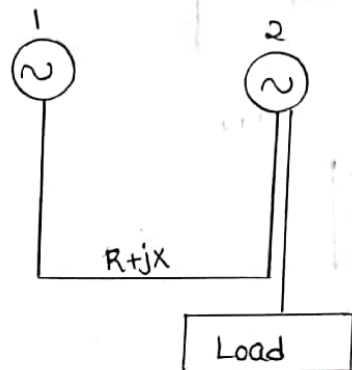
$$C_2 = 587.5 \text{ RS/hr}$$

$$\text{Saving in cost} = C_2 - C_1$$

$$= -22.5 \text{ RS/hr}$$

Optimal load dispatch including transmission losses (or)

optimal generation allocation including transmission loss:



Two identical generators connected to a transmission line.

Consider two identical generators with equal incremental production cost. The two generators have to supply the total load and each generator supplies half of the load. The generator-2 is more economical to supply the load because generator-1 has to supply load as well as losses.

Economic dispatch including losses is defined as

$$\min F_t = \sum_{n=1}^N F_n$$

$$P_D + P_L = \sum_{n=1}^N P_n$$

By using Lagrangian's multiplier, the auxiliary equation is

$$F = F_t + \lambda (P_D + P_L - \sum_{n=1}^N P_n)$$

differentiating above eqn w.r.to  $P_n$  & equating it to zero

$$\frac{dF}{dP_n} = \frac{dF_t}{dP_n} + \lambda \left( \frac{\partial P_L}{\partial P_n} - 1 \right) = 0$$

$$\frac{dF}{dP_n} = \frac{dF_t}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda$$

$$\frac{dF_t}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda$$

$$F_t = F_1 + F_2 + \dots + F_n$$

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda$$

Where

$\lambda$  = cost of received power

$\frac{dF_n}{dP_n}$  = Incremental fuel cost of nth unit

$\frac{\partial P_L}{\partial P_n}$  = Incremental transmission loss of nth unit

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda$$

$$\frac{dF_n}{dP_n} = \lambda \left( 1 - \frac{\partial P_L}{\partial P_n} \right)$$

$$\lambda = \frac{(dF_n/dP_n)}{\left( 1 - \frac{\partial P_L}{\partial P_n} \right)}$$

$\frac{\partial P_L}{\partial P_n}$  = Incremental transmission loss

$$\text{Penalty factor } L_n = \frac{1}{1 - \frac{\partial P_L}{\partial P_n}}$$

$$\lambda = \frac{dF_n}{dP_n} L_n$$



$$L_1 \frac{dF_1}{dP_1} = L_2 \frac{dF_2}{dP_2} = \dots = L_n \frac{dF_n}{dP_n} = \lambda$$

Power loss is expressed as

$$P_L = \sum_m \sum_n P_m B_{mn} P_n$$

$P_m$  and  $P_n \rightarrow$  source loading

$B_{mn} \rightarrow$  Transmission loss coefficient

$$B_{mn} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$\frac{dP_L}{dP_n} = \frac{dP_L}{dP_m}$$

$$\text{i.e. } P_m = P_n$$

$$P_L = \sum_m P_m^2 B_{mn}$$

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2P_m B_{mn} = 2 \sum_m P_m B_{mn}$$

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \rightarrow \textcircled{1}$$

we know that

$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n$$

Substitute above value and  $\frac{\partial P_L}{\partial P_n}$  in eq  $\textcircled{1}$

$$F_{nn} P_n + f_n + \lambda \cdot 2 \sum_m P_m B_{mn} = \lambda$$

$$F_{nn} P_n + f_n + 2\lambda \left[ \sum_{m=n} P_m B_{mn} + \sum_{m \neq n} P_m B_{mn} \right] = \lambda$$

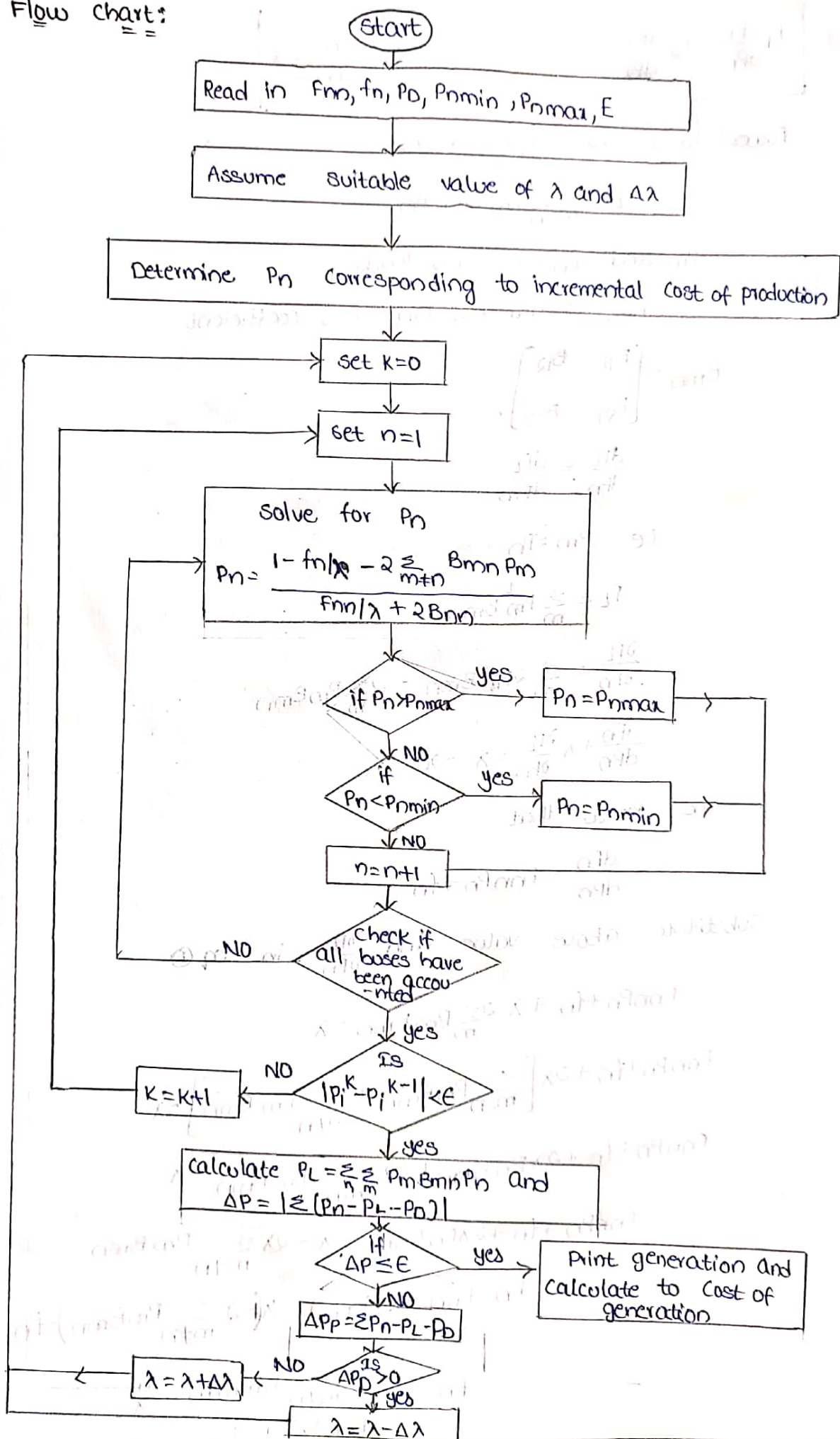
$$F_{nn} P_n + f_n + 2\lambda P_n B_{nn} + 2\lambda \sum_{m \neq n} P_m B_{mn} = \lambda$$

$$F_{nn} P_n + f_n + 2\lambda P_n B_{nn} = \lambda - 2\lambda \sum_{m \neq n} P_m B_{mn}$$

$$P_n (F_{nn} + 2\lambda B_{nn}) = \lambda \left( 1 - 2 \sum_{m \neq n} P_m B_{mn} \right) - f_n$$

$$P_n = \frac{1 - 2 \sum_{m \neq n} B_{mn} P_m - \frac{f_n}{\lambda}}{F_{nn}/\lambda + 2B_{nn}}$$

# Flow chart:



This process is time consuming and we use following iterative procedure.

### Algorithm:

- Assume a suitable value of  $\lambda$
- Compute individual generations with respect to incremental production cost.
- Calculate the generations at all the buses.

$$P_n = \frac{1 - f_n/\lambda - 2 \sum_{m \neq n} B_{mn} P_m}{\frac{f_{nn}}{\lambda} + 2B_{nn}}$$

- Check if the difference in power at all the buses less than prespecified value.
- If not go back to step-3.
- Calculate losses and changes in power

$$P_L = \sum_n \sum_m B_{mn} P_m P_n$$

$$\Delta P = |\sum (P_n - P_L - P_D)|$$

- If  $\Delta P$  is less than  $\epsilon$ , then print generation determine cost of generation, otherwise obtain the value of  $\lambda$  and go back to step-3.

- On a system consisting of two generating plants the incremental cost is  $\frac{dc_1}{dP_{G1}} = 0.008P_{G1} + 8$ ,  $\frac{dc_2}{dP_{G2}} = 0.012P_{G2} + 9$ . The system is operating on economical dispatch with  $P_{G1} = P_{G2} = 500\text{MW}$ ,  $\frac{\partial P_L}{\partial P_{G2}} = 0.2$ . Find the penalty factor of plant-1.

Sol:

Given,

IFC of two units

$$\frac{dc_1}{dP_{G1}} = 0.008P_{G1} + 8$$

$$\frac{dc_2}{dP_{G2}} = 0.012P_{G2} + 9$$

Economic dispatch,  $P_{G1} = P_{G2} = 500\text{MW}$

Incremental transmission loss of 2,

$$\frac{\partial P_L}{\partial P_{G2}} = 0.2$$

$$\text{Penalty factor } L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G2}}}$$

Economic load dispatch with losses

$$L_1 \frac{dc_1}{dP_{G1}} = L_2 \frac{dc_2}{dP_{G2}} = \lambda$$

Penalty factor of plant-2,

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G2}}} = \frac{1}{1 - 0.2} = 1.25$$

Penalty factor of plant-1,

$$\begin{aligned} L_1 &= \frac{L_2 \frac{dc_2}{dP_{G2}}}{\frac{dc_1}{dP_{G1}}} \\ &= \frac{1.25 (0.012P_{G2} + 9)}{0.008P_{G1} + 8} \\ &= \frac{1.25 (0.012(500) + 9)}{0.008(500) + 8} \end{aligned}$$

$$L_1 = 1.56$$



The IFC of two plants are  $\frac{dc_1}{dP_{G1}} = 0.075P_{G1} + 18$  RS/MWhr,

$\frac{dc_2}{dP_{G2}} = 0.08P_{G2} + 16$  RS/MWhr. The loss coefficient are given as  $B_{11} = 0.0015$  per MW,

$B_{12} = -0.0004$  per MW and  $B_{22} = 0.0032$  per MW for  $\lambda = 25$  RS/MWhr. Find the real power generation, total load demand, transmission power loss.

Sol

Given IFC are,

$$\frac{dc_1}{dP_{G1}} = 0.075P_{G1} + 18 \text{ RS/MWhr}$$

$$\frac{dc_2}{dP_{G2}} = 0.08P_{G2} + 16 \text{ RS/MWhr}$$

$$\text{Loss of coefficients} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 0.0015 & -0.0004 \\ 0.0004 & 0.0032 \end{bmatrix} \text{ per MW}$$

$$\lambda = 25 \text{ RS/MWhr}$$

$$\text{Power loss} = \sum_{m=1}^2 \sum_{n=1}^2 P_m B_{mn} P_n$$

$$P_L = P_{G1}^2 B_{11} + P_{G2}^2 B_{22} + 2P_{G1}P_{G2}B_{12}$$

Incremental loss of plant - 1

$$\frac{\partial P_L}{\partial P_{G1}} = 2P_{G1}B_{11} + 2P_{G2}B_{12}$$

$$= 2P_{G1}(0.0015) + 2P_{G2}(-0.0004)$$

$$= 0.003P_{G1} - 0.0008P_{G2}$$

$$\text{Penalty factor} = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

$$L_1 = \frac{1}{1 - 0.003 P_{G1} + 0.0008 P_{G2}}$$

Incremental loss of plant-2,

$$\begin{aligned} \frac{\partial P_L}{\partial P_{G2}} &= 2 P_{G2} B_{22} + 2 P_{G1} B_{12} \\ &= 2 P_{G2} (0.0032) + 2 P_{G1} (-0.0004) \\ &= 0.0064 P_{G2} - 0.0008 P_{G1} \end{aligned}$$

Penalty factor of plant-2

$$L_2 = \frac{1}{1 - 0.0064 P_{G2} + 0.0008 P_{G1}}$$

condition for economic scheduling with losses

$$L_1 \frac{dc_1}{dP_{G1}} = L_2 \frac{dc_2}{dP_{G2}} = \lambda$$

$$\Rightarrow L_1 \frac{dc_1}{dP_{G1}} = \lambda$$

$$\frac{1}{1 - 0.003 P_{G1} + 0.0008 P_{G2}} (0.075 P_{G1} + 18) = 25$$

$$0.075 P_{G1} + 18 = 25 - 0.075 P_{G1} + 0.02 P_{G2}$$

$$0.15 P_{G1} - 0.02 P_{G2} = 7 \rightarrow (1)$$

$$\Rightarrow L_2 \frac{dc_2}{dP_{G2}} = \lambda$$

$$\frac{1}{1 - 0.0064 P_{G2} + 0.0008 P_{G1}} (0.08 P_{G2} + 16) = 25$$

$$0.08 P_{G2} + 16 = 25 - 0.16 P_{G2} + 0.02 P_{G1}$$

$$-0.02P_{G1} + 0.24P_{G2} = 9 \rightarrow (2)$$

solving eq. ① & eq. ②

$$P_{G1} = 52.24 \text{ MW}$$

$$P_{G2} = 41.85 \text{ MW}$$

Power loss  $P_L = P_{G1}^2 B_{11} + P_{G2}^2 B_{22} + 2P_{G1}P_{G2}B_{12}$

$$P_L = (52.24)^2(0.0015) + (41.85)^2(0.0032) + 2 \times 52.24 \times 41.85(-0.0004)$$

$$P_L = 7.95 \text{ MW}$$

Total load demand  $P_D = P_{G1} + P_{G2} - P_L$

$$= 52.24 + 41.85 - 7.95$$

$$P_D = 86.14 \text{ MW}$$

- The cost curve of two plants are  $C_1 = 0.05P_{G1}^2 + 20P_{G1} + 150 \text{ Rs/hr}$ ,  $C_2 = 0.05P_{G2}^2 + 15P_{G2} + 180 \text{ Rs/hr}$ . Loss of coefficients are  $B_{11} = 0.0015 \text{ per MW}$ ,  $B_{12} = B_{21} = -0.0004 \text{ per MW}$ ,  $B_{22} = 0.0032 \text{ per MW}$ . Determine economical generation scheduling with  $\lambda = 30 \text{ Rs/MWhr}$  and corresponding system load that can be met with if the total load is connected to the system is 120 MW. Taking 4% change in the value of  $\lambda$ . What should be the value of  $\lambda$  in next iteration.

sol:

Given,

$$C_2 = 0.05P_{G2}^2 + 15P_{G2} + 180 \text{ Rs/hr}$$

$$C_1 = 0.05P_{G1}^2 + 20P_{G1} + 150 \text{ Rs/hr}$$



$$\frac{dc_1}{dP_{G1}} = 0.1P_{G1} + 20$$

$$\frac{dc_2}{dP_{G2}} = 0.1P_{G2} + 15$$

$$B = \begin{bmatrix} 0.0015 & -0.0004 \\ -0.0004 & 0.0032 \end{bmatrix} \text{ per MW}$$

$$\lambda = 30 \text{ Rs/MWhr}$$

$$\text{power loss} = \sum_{m=1}^2 \sum_{n=1}^2 P_m B_{mn} P_n$$

$$P_L = P_{G1}^2 B_{11} + P_{G2}^2 B_{22} + 2P_{G1} P_{G2} B_{12}$$

Incremental loss of plant-1,

$$\frac{\partial P_L}{\partial P_{G1}} = 2P_{G1} B_{11} + 2P_{G2} B_{12}$$

$$= 2P_{G1}(0.0015) + 2P_{G2}(-0.0004)$$

$$= 0.003P_{G1} - 0.0008P_{G2}$$

$$\text{Penalty factor } L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

$$= \frac{1}{1 - 0.003P_{G1} + 0.0008P_{G2}}$$

Incremental cost of plant-2,

$$\frac{\partial P_L}{\partial P_{G2}} = 2P_{G2} B_{22} + 2P_{G1} B_{12}$$

$$= 2P_{G2}(0.0032) + 2P_{G1}(-0.0004)$$

$$= 0.0064P_{G2} - 0.0008P_{G1}$$



Penalty factor  $L_2 = \frac{1}{1 - \frac{\partial PL}{\partial PG_2}}$

$$= \frac{1}{1 - 0.0064 PG_2 + 0.0008 PG_1}$$

condition for economic scheduling with losses

$$L_1 \frac{dC_1}{dPG_1} = L_2 \frac{dC_2}{dPG_2} = \lambda$$

$$\Rightarrow L_1 \frac{dC_1}{dPG_1} = \lambda$$

$$\frac{1}{1 - 0.003 PG_1 + 0.0008 PG_2} (0.1 PG_1 + 20) = 30$$

$$0.1 PG_1 + 20 = 30 - 0.09 PG_1 + 0.024 PG_2$$

$$0.19 PG_1 - 0.024 PG_2 = 100 \rightarrow \textcircled{1}$$

$$\Rightarrow L_2 \frac{dC_2}{dPG_2} = \lambda$$

$$\frac{1}{1 - 0.0064 PG_2 + 0.0008 PG_1} (0.1 PG_2 + 15) = 30$$

$$0.292 PG_2 - 0.024 PG_1 = 15 \rightarrow \textcircled{2}$$

solving eq ① & ②

$$PG_1 = 59.74 \text{ MW}$$

$$PG_2 = 56.28 \text{ MW}$$

$$P_L = PG_1^2 B_{11} + PG_2^2 B_{22} + 2 PG_1 PG_2 B_{12}$$

$$= (59.74)^2 (0.0015) + (56.28)^2 (0.0032) + 2(59.74)(56.28)(-0.0004)$$

$$P_L = 12.79 \text{ MW}$$

Total load demand,  $P_D = P_{G1} + P_{G2} - P_L$

$$= 59.74 + 56.78 - 12.79$$

$$P_D = 103.2215 \text{ MW}$$

$$\text{Total load} = 120 \text{ MW}$$

$$\text{change in load} = 120 - (59.74 + 56.28)$$

$$= 39 \text{ MW}$$

$$\lambda' = \frac{4}{100} \times 30$$

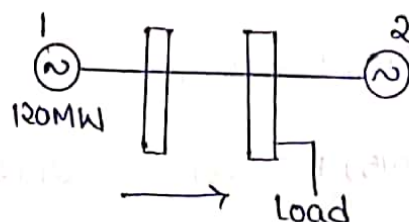
$$\lambda' = 1.2$$

$$\text{change in } \lambda \text{ value in next iteration} = 30 - 1.2$$

$$= 28.8 \text{ RS/MWhr.}$$

- A two bus system consisting of two power plants connected by a transmission line as shown in fig.  $C_1 = 0.015 P_{G1}^2 + 18 P_{G1} + 20 \text{ RS/MWhr}$ ,  $C_2 = 0.03 P_{G2}^2 + 22 P_{G2} + 40 \text{ RS/MWhr}$ . When a power of 120 MW is transmitted from plant '1' to the load, a loss of 16.424 MW is incurred. Determine the optimal scheduling of the plant, load demand if cost of received power is 26 RS/MWhr. solve the problem while using coordination equations, penalty factor method.

Sol:



Given,

$$C_1 = 0.015 P_{G1}^2 + 18 P_{G1} + 20 \text{ Rs/MWhr}$$

$$C_2 = 0.03 P_{G2}^2 + 22 P_{G2} + 40 \text{ Rs/MWhr}$$

$$P_L = 16.425 \text{ MW}$$

$$P_{G1} \text{ (transmitted power)} = 120 \text{ MW}$$

$$P_L = P_{G1}^2 B_{11} + P_{G2}^2 B_{22} + 2 P_{G1} P_{G2} B_{12}$$

Since Load is connected nearer to plant-2

Loss coefficient of plant-2,  $B_{12} = B_{21} = B_{22} = 0$

$$P_L = P_{G1}^2 B_{11}$$

$$B_{11} = \frac{P_L}{P_{G1}^2} = \frac{16.425}{(120)^2}$$

$$B_{11} = 0.0011 / \text{MW}$$

$$P_L = P_{G1}^2 B_{11}$$

$$\begin{aligned} \frac{\partial P_L}{\partial P_{G1}} &= 2 P_{G1} B_{11} \\ &= 2 P_{G1} \times 0.0011 \end{aligned}$$

$$= 0.0022 P_{G1}$$

$$L_1 = \frac{1}{1 - 0.0022 P_{G1}}$$

$$\frac{\partial P_L}{\partial P_{G2}} = 0$$

$$L_2 = \frac{1}{1 - 0} = 1$$

Penalty factor method:

$$L_1 \frac{dc_1}{dP_{G1}} = \lambda$$

$$\frac{dc_1}{dP_{G1}} = 0.015 \times 2P_{G1} + 18$$

$$= 0.03P_{G1} + 18$$

$$\frac{1}{1 - 0.0022P_{G1}} (0.03P_{G1} + 18) = 26$$

$$P_{G1} = 91.74 \text{ MW}$$

$$L_2 \frac{dc_2}{dP_{G2}} = \lambda$$

$$\frac{dc_2}{dP_{G2}} = 0.03 \times 2P_{G2} + 22$$

$$= 0.06P_{G2} + 22$$

$$1(0.06P_{G2} + 22) = 26$$

$$P_{G2} = \frac{4}{0.06} = 66.6 \text{ MW}$$

$$P_L = P_{G1}^2 B_{11} = (91.74)^2 \times 0.0011$$

$$P_L = 9.25 \text{ MW}$$

Load demand  $P_D = P_{G1} + P_{G2} - P_L$

$$= 91.74 + 66.6 - 9.25$$

$$= 149.09 \text{ MW}$$



Two power plants are connected together a transmission line and load at plant 2 shown in fig. when 100MW is transmitted from plant-1, the transmission loss is 10MW. Cost characteristics are  $C_1 = 0.05 P_{G1}^2 + 13 P_{G1}$  Rs/MWhr,  $C_2 = 0.06 P_{G2}^2 + 12 P_{G2}$  Rs/MWhr. Find optimal scheduling for  $\lambda = 22$ .

Sol:

Given,

$$C_1 = 0.05 P_{G1}^2 + 13 P_{G1}$$

$$C_2 = 0.06 P_{G2}^2 + 12 P_{G2}$$

$$P_L = 10 \text{ MW}$$

$$P_{G1} \text{ (Transmitted power)} = 100 \text{ MW}$$

$$P_L = P_{G1}^2 B_{11} + P_{G2}^2 B_{22} + 2 P_{G1} P_{G2} B_{12}$$

Since load is connected at plant-2

loss of coefficients of plant-2,  $B_{12} = B_{21} = B_{22} = 0$

$$P_L = P_{G1}^2 B_{11}$$

$$B_{11} = \frac{P_L}{P_{G1}^2} = \frac{10}{(100)^2}$$

$$B_{11} = 0.001 \text{ per MW}$$

$$P_L = P_{G1}^2 B_{11}$$

$$\frac{\partial P_L}{\partial P_{G1}} = 2 P_{G1} B_{11}$$

$$= 2 P_{G1} \times 0.001$$

$$= 0.002 P_{G1}$$

$$\frac{\partial P_L}{\partial P_{G2}} = 0$$

$$\frac{dc_1}{dP_{G1}} = 2P_{G1}(0.05) + 13$$

$$= 0.1P_{G1} + 13$$

$$\frac{dc_2}{dP_{G2}} = 2P_{G2}0.06 + 12$$

$$= 0.12P_{G2} + 12$$

Coordination equation method:

$$\lambda = 22$$

$$\frac{dc_1}{dP_{G1}} + \lambda \frac{\partial P_L}{\partial P_{G1}} = \lambda$$

$$0.1P_{G1} + 13 + 22(0.002P_{G1}) = 22$$

$$0.144P_{G1} = 9$$

$$P_{G1} = 62.5 \text{ MW}$$

$$\frac{dc_2}{dP_{G2}} = 0.12P_{G2} + 12$$

$$\frac{dc_2}{dP_{G2}} + \lambda \frac{\partial P_L}{\partial P_{G2}} = \lambda$$

$$0.12P_{G2} + 12 + 22(0) = 22$$

$$0.12P_{G2} = 10$$

$$P_{G2} = 83.33 \text{ MW}$$

$$P_L = P_{G1}^2 B_{11}$$

$$= (62.5)^2 (0.001)$$

$$= 3.9 \text{ MW}$$

$$P_D = P_{G1} + P_{G2} - P_L = 141.9 \text{ MW}$$

A power system operates an economic load dispatch with  $\lambda = 60 \text{ Rs/MWhr}$ . If raising the output of plant-2 while 100kW (while the other output is kept constant) results in increased power loss of 12kW for the system. what is the approximate additional cost for hour if the output of the plant is increased by 1MW.

Sol:

Given,

$$\lambda = 60 \text{ Rs/MWhr}$$

Incremental output of plant-2,  $P_{G2} = 100 \text{ kW}$

$$\Delta P_L = 12 \text{ kW}$$

Additional cost  $dc_2 = ?$

$$dP_{G2} = 1 \text{ MW}$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G2}}}$$

$$= \frac{1}{1 - \frac{12}{100}}$$

$$L_2 = 1.136$$

$$L_2 \frac{dc_2}{dP_{G2}} = \lambda$$

$$\frac{dc_2}{dP_{G2}} = \frac{\lambda}{L_2}$$

$$\frac{dc_2}{dP_{G2}} = \frac{60}{1.136}$$

$$\frac{dc_2}{dP_{G2}} = 52.81 \text{ Rs/MWhr}$$

Additional cost for 1MW dp

$$dc_2 = 52.81 \times dP_{G2}$$

$$= 52.81 \times 1 \text{ MW} \times \frac{\text{Rs}}{\text{MWhr}}$$

$$dc_2 = 52.81 \text{ Rs/hr}$$

- Two thermal plants are interconnected to supply

$$\frac{dc_1}{dP_{G1}} = 20 + 10P_{G1} \text{ Rs/MWhr}, \frac{dc_2}{dP_{G2}} = 15 + 10P_{G2} \text{ Rs/MWhr}. P_{G1} \text{ and } P_{G2} \text{ are}$$

expressed in per unit in 100MVA base. The transmission loss

$$P_L = 0.1P_{G1}^2 + 0.2P_{G2}^2 + 0.1P_{G1}P_{G2} \text{ p.u.}, \lambda = 15 \text{ Rs/MWhr. Find the optimal generations.}$$

Sol:

Given,

$$\frac{dc_1}{dP_{G1}} = 20 + 10P_{G1} \text{ Rs/MWhr}$$

$$\frac{dc_2}{dP_{G2}} = 15 + 10P_{G2} \text{ Rs/MWhr}$$

$$P_L = 0.1P_{G1}^2 + 0.2P_{G2}^2 + 0.1P_{G1}P_{G2}$$

$$\lambda = 15 \text{ Rs/MWhr}$$

$$\frac{\partial P_L}{\partial P_{G1}} = 0.2P_{G1} + 0.1P_{G2}$$

$$\frac{\partial P_L}{\partial P_{G2}} = 0.4P_{G2} + 0.1P_{G1}$$



Coordination equation method:

$$\frac{dc_1}{dP_{G1}} + \lambda \frac{\partial P_L}{\partial P_{G1}} = \lambda$$

$$20 + 10P_{G1} + 50(0.2P_{G1} + 0.1P_{G2}) = 50$$

$$20 + 10P_{G1} + 10P_{G1} + 5P_{G2} = 50$$

$$20P_{G1} + 5P_{G2} = 30 \rightarrow (1)$$

$$\frac{dc_2}{dP_{G2}} + \lambda \frac{\partial P_L}{\partial P_{G2}} = \lambda$$

$$15 + 10P_{G2} + 50(0.4P_{G2} + 0.1P_{G1}) = 50$$

$$20P_{G2} + 5P_{G1} = 35 \rightarrow (2)$$

solving eq (1) & eq (2)

$$P_{G1} = 1.26 \text{ PU}$$

$$P_{G2} = 0.956 \text{ PU}$$

To get in MW per unit is multiplied with 100MW base

$$P_{G1} = 1.26 \times 100 = 126 \text{ MW}$$

$$P_{G2} = 0.956 \times 100 = 95.6 \text{ MW}$$

## Loss coefficients (or) Bmn coefficients:

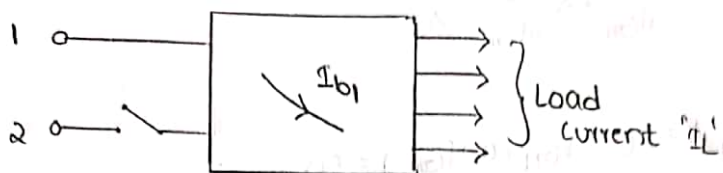


Fig-a

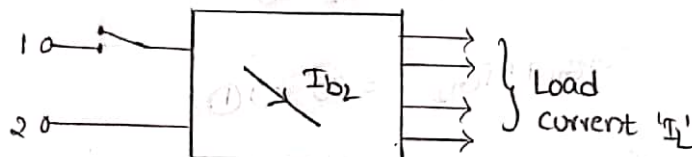


Fig-b

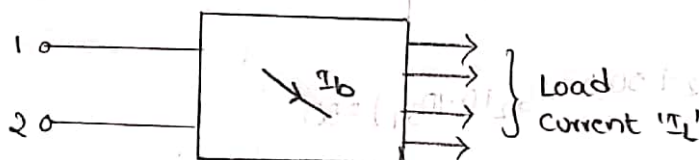


Fig-c

Schematic diagram of two generators and various loads with considering one branch of line 'b'.

Consider a 3- $\phi$  line 'b', the total load current supplied by generator-1 then current in the line will be

$$I_{b1}, \alpha_{b1} = \frac{I_{b1}}{I_b}$$

when generator-1 is open and generator-2 alone supply the load current, then current in the line will be  $I_{b2}$ ,

$$\alpha_{b2} = \frac{I_{b2}}{I_b}$$

When both generators are excited to supply the load current then the current will be  $I_b$ .

According to superposition principle  $I_b = \alpha_{b1} I_1 + \alpha_{b2} I_2$

Where,  $I_1$  and  $I_2$  are currents of plant-1 and plant-2

$\alpha_{b1}$  and  $\alpha_{b2}$  are current distribution factors

### Assumptions:

- $\frac{X}{R}$  ratio must be same for the transmission line.
- phase angle of all the load currents must be same.
- Current distributions factors are real.

$$I_1 = |I_1| \angle \theta_1$$

$$I_2 = |I_2| \angle \theta_2$$

$$I_1 = |I_1| [\cos \theta_1 + j \sin \theta_1]$$

$$I_2 = |I_2| [\cos \theta_2 + j \sin \theta_2]$$

$$I_b = \alpha_{b1} |I_1| (\cos \theta_1 + j \sin \theta_1) + \alpha_{b2} |I_2| (\cos \theta_2 + j \sin \theta_2)$$

$$I_b = \alpha_{b1} |I_1| \cos \theta_1 + j \alpha_{b1} |I_1| \sin \theta_1 + \alpha_{b2} |I_2| \cos \theta_2 + j |I_2| \alpha_{b2} \sin \theta_2$$

$$I_b = \alpha_{b1} |I_1| \cos \theta_1 + \alpha_{b2} |I_2| \cos \theta_2 + j [\alpha_{b1} |I_1| \sin \theta_1 + \alpha_{b2} |I_2| \sin \theta_2]$$

squaring on both sides

$$I_b^2 = \alpha_{b1}^2 |I_1|^2 \cos^2 \theta_1 + \alpha_{b2}^2 |I_2|^2 \cos^2 \theta_2 + 2 \alpha_{b1} \alpha_{b2} |I_1| |I_2| \cos \theta_1 \cos \theta_2$$

$$+ 2 \alpha_{b1} \alpha_{b2} |I_1| |I_2| \sin \theta_1 \sin \theta_2 + \alpha_{b1}^2 |I_1|^2 \sin^2 \theta_1 + \alpha_{b2}^2 |I_2|^2 \sin^2 \theta_2$$

$$I_b^2 = \alpha_{b1}^2 |I_1|^2 + \alpha_{b2}^2 |I_2|^2 + 2 \alpha_{b1} \alpha_{b2} |I_1| |I_2| \cos(\theta_1 - \theta_2)$$

$$\text{Power loss} = \sum_{b=1}^n 3 I_b^2 R_b$$

$R_b \rightarrow$  Branch resistance

$$|I_1| = \frac{P_1}{\sqrt{3} |V_1| \cos \phi_1}, \quad |I_2| = \frac{P_2}{\sqrt{3} |V_2| \cos \phi_2}$$

$|V_1|$  &  $|V_2|$  are bus voltages

$$\alpha_{b1}^2 |I_1|^2 = \alpha_{b1}^2 \frac{P_1^2}{3|V_1|^2 \cos^2 \phi_1}$$

$$\alpha_{b2}^2 |I_2|^2 = \alpha_{b2}^2 \frac{P_2^2}{3|V_2|^2 \cos^2 \phi_2}$$

$$P_L = \sum_{b=1}^b \frac{3 P_1^2 \alpha_{b1}^2 R_b}{3 |V_1|^2 \cos^2 \phi_1} + \sum_{b=1}^b \frac{3 \alpha_{b2}^2 \cdot R_b \cdot P_2^2}{3 |V_2|^2 \cos^2 \phi_2} + 2 \sum_{b=1}^b \frac{3 \alpha_{b1} \alpha_{b2} P_1 P_2 R_b \cos(\theta_1 - \theta_2)}{3 |V_1| \cos \phi_1 \cdot 3 |V_2| \cos \phi_2}$$

$$P_L = \sum_{b=1}^b \frac{P_1^2 \alpha_{b1}^2 R_b}{|V_1|^2 \cos^2 \phi_1} + \sum_{b=1}^b \frac{\alpha_{b2}^2 R_b P_2^2}{|V_2|^2 \cos^2 \phi_2} + \sum_{b=1}^b \frac{2 \alpha_{b1} \alpha_{b2} P_1 P_2 R_b \cos(\theta_1 - \theta_2)}{|V_1| |V_2| \cos \phi_1 \cos \phi_2}$$

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + 2 P_1 P_2 B_{12}$$

$$B_{11} = \sum_{b=1}^b \frac{\alpha_{b1}^2 R_b}{|V_1|^2 \cos^2 \phi_1}$$

$$B_{22} = \sum_{b=1}^b \frac{\alpha_{b2}^2 R_b}{|V_2|^2 \cos^2 \phi_2}$$

$$B_{12} = B_{21} = \sum_{b=1}^b \frac{\alpha_{b1} \alpha_{b2} R_b \cos(\theta_1 - \theta_2)}{|V_1| |V_2| \cos \phi_1 \cdot \cos \phi_2}$$

If there are 'm' and 'n' sources

$$B_{mm} = \sum_{b=1}^b \frac{\alpha_{bm}^2 R_b}{|V_m|^2 \cos^2 \phi_m}, B_{nn} = \sum_{b=1}^b \frac{\alpha_{bn}^2 R_b}{|V_n|^2 \cos^2 \phi_n}$$

$$B_{mn} = \sum_{b=1}^b \frac{\alpha_{bm} \alpha_{bn} R_b \cos(\theta_m - \theta_n)}{|V_m| |V_n| \cos \phi_m \cdot \cos \phi_n}$$



Exact transmission loss formula:

Let  $S_i$  be the injected power at bus 'i'

$$= \left( \begin{array}{c} \text{Total generated} \\ \text{power at bus 'i'} \end{array} \right) - (\text{load at bus 'i'})$$

The summation of all the powers at all the buses gives total losses of a system.

$$P_L + jQ_L = \sum_{i=1}^n S_i = \sum_{i=1}^n V_i I_i^* = V_{bus}^T I_{bus}^*$$

$V_{bus}$  &  $I_{bus}$  → bus voltage and bus current

$P_L$  &  $Q_L$  → Active and reactive power loss

$$V_{bus} = I_{bus} Z_{bus}$$

$$P_L + jQ_L = (I_{bus} Z_{bus})^T I_{bus}^*$$

$$= Z_{bus}^T I_{bus}^T I_{bus}^*$$

$Z_{bus}$  → bus impedance matrix

$Z_{bus}$  → symmetric matrix ( $A^T = A$ )

$$Z_{bus}^T = Z_{bus}$$

$$P_L + jQ_L = Z_{bus} I_{bus}^T I_{bus}^*$$

$$Z_{bus} = R + jX$$

$$= \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \dots & r_{nn} \end{bmatrix} + j \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nn} \end{bmatrix}$$

$$I_{bus} = I_p + jI_q$$

$$= \begin{bmatrix} I_{p1} \\ I_{p2} \\ \vdots \\ I_{pn} \end{bmatrix} + j \begin{bmatrix} I_{q1} \\ I_{q2} \\ \vdots \\ I_{qn} \end{bmatrix}$$

$$P_L + jQ_L = (R + jX)(I_p + jI_q)^T (I_p - jI_q)$$

$$= (R + jX)(I_p^T + jI_q^T)(I_p - jI_q)$$

$$= (RI_p^T + jRI_q^T + jXI_p^T - XI_q^T)(I_p - jI_q)$$

$$= RI_p^T I_p + jRI_q^T I_p + jXI_p^T I_p - XI_q^T I_p - jRI_p^T I_q + RI_q I_q^T + XI_p^T I_q + jXI_q I_q^T$$

$$P_L + jQ_L = RI_p I_p^T + RI_q I_q^T + XI_q I_p^T - XI_p I_q^T + j(RI_p I_q^T + XI_p I_p^T + XI_q I_q^T - RI_p^T I_q)$$

$$P_L = RI_p I_p^T + RI_q I_q^T + XI_q I_p^T - XI_p I_q^T$$

since 'X' is a symmetric matrix

$$XI_p^T I_q = XI_q^T I_p \quad (XA^T B = XB^T A)$$

$$P_L = RI_p I_p^T + RI_q I_q^T + XI_q I_p^T - XI_p I_q^T$$

$$P_L = RI_p I_p^T + RI_q I_q^T$$

$$= \sum_{j=1}^n \sum_{k=1}^n r_{jk} (I_{pj} I_{pk} + I_{qj} I_{qk})$$

Above equation is represented in terms of only bus currents. so we use bus voltages and bus powers to determine power loss.

For bus 'i'

$$P_i + jQ_i = V_i I_i^* = V_i (I_{P_i} - jI_{Q_i})$$

$$= |V_i| (\cos \delta_i + j \sin \delta_i) (I_{P_i} - jI_{Q_i}) \rightarrow \textcircled{A}$$

$$P_i + jQ_i = |V_i| (\cos \delta_i I_{P_i} - j \cos \delta_i I_{Q_i} + j \sin \delta_i I_{P_i} + \sin \delta_i I_{Q_i})$$

separating real and ~~com~~ imaginary parts

$$P_i = |V_i| \cos \delta_i I_{P_i} + |V_i| \sin \delta_i I_{Q_i} \rightarrow \textcircled{1}$$

$$Q_i = |V_i| \sin \delta_i I_{P_i} - |V_i| \cos \delta_i I_{Q_i} \rightarrow \textcircled{2}$$

Multiply eq. ① by  $\sin \delta_i$

eq. ② by  $\cos \delta_i$

$$P_i \sin \delta_i = I_{P_i} |V_i| \cos \delta_i \sin \delta_i + |V_i| I_{Q_i} \sin^2 \delta_i$$

$$\begin{array}{r} Q_i \cos \delta_i = I_{P_i} |V_i| \sin \delta_i \cos \delta_i - |V_i| I_{Q_i} \cos^2 \delta_i \\ \hline - \qquad \qquad \qquad + \end{array}$$

$$P_i \sin \delta_i - Q_i \cos \delta_i = |V_i| I_{Q_i}$$

$$I_{Q_i} = \frac{1}{|V_i|} (P_i \sin \delta_i - Q_i \cos \delta_i)$$

$$I_{P_i} = \frac{1}{|V_i|} (P_i \cos \delta_i + Q_i \sin \delta_i)$$

Substitute  $I_{p_i}, I_{q_i}$  values in eq (A)

$$P_L = \sum_{j=1}^n \sum_{k=1}^n \alpha_{jk} (P_j P_k + Q_j Q_k) + B_{jk} (Q_j P_k - P_j Q_k)$$

$$\alpha_{jk} = \frac{r_{jk}}{|v_j| |v_k|} \cos(\delta_j - \delta_k)$$

$$B_{jk} = \frac{r_{jk}}{|v_j| |v_k|} \sin(\delta_j - \delta_k)$$



## Unit-2 Hydro Thermal Scheduling

### Introduction:

Our power system is a large interconnected network with various sources of energy like hydro, thermal are operated in such a way that cost of generation must be minimum.

The Capital Cost of thermal plant is smaller than hydro plant. While running cost of hydro plant is practically negligible and thermal plant is having more running cost.

For a particular load duration curve thermal plant acts like base load plant and hydro plant acts like peak load plant.

The cost of operation of hydro thermal system is the cost of fuel in thermal plants and cost of water in hydro plants. Hence cost of generation must be minimum. So that transmission losses is also minimum for a particular load.

The water head and level of water must be constant over a specific period of time.

## Optimal Scheduling of Hydro Thermal System

Scheduling is the process of allocation of generation from various generating units. It is one of the cost effective mode of technique so that total cost of generation must be minimum.

In case of thermal plants optimal scheduling can be determined at any instant without referring to other instants.

So it is called static optimization.

In hydro plants optimal scheduling depends on water availability for a particular period of time so it is called dynamic optimization.

The operation of hydro thermal system is very complex since hydro plants are having negligible operating cost. The cost of hydro thermal system is the cost of fuel in thermal plants under constraints of water availability for hydro generation over a specific period of time.



## Storage of Reservoir:

\* Storage of reservoir must be specified at the beginning and ending.

\* Water inflow and load demand are always function of time.

\* Determine water discharge  $Q(t)$  so that to minimize cost of thermal generation.

$$C_T = \int_0^T C'(P_{GT}(t))$$

$$C_H = \int_0^T C'(P_{GH}(t))$$

\* Meeting the load demand

$$P_D = P_{GT}(t) + P_{GH}(t) - P_L(t)$$

$$P_{GT}(t) + P_{GH}(t) - P_L(t) - P_D(t) = 0$$

The above equation is called power balance eq.

\* Water availability

$$X'(T) - X'(0) = - \int_0^T J(t) dt + \int_0^T q(t) dt = 0$$

where  $X'(T) - X'(0)$  are specified water storages at the ending & beginning.

$J(t) \rightarrow$  water inflow

$q(t) \rightarrow$  water discharge

## Hydro Generation:

$P_{GH}(t)$  is always a function of water discharge and specified water storage at ending.

$$P_{GH}(t) = F(q(t) \times h(t))$$

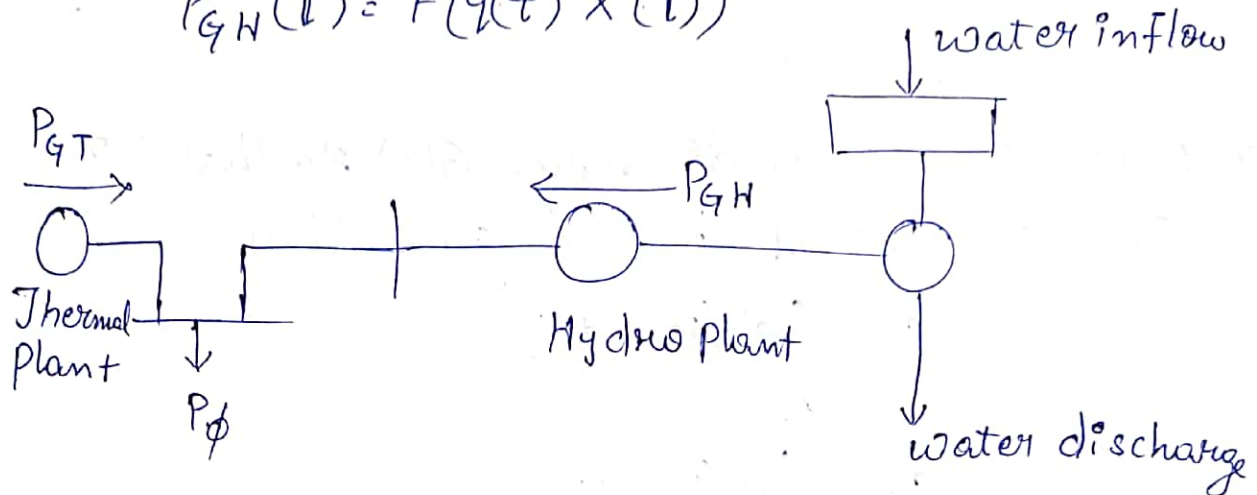


Fig: Fundamental Hydro thermal System

Water storage is a dependent variable and water discharge is a independent variable for hydro power generations.

1) A Constant load of 300MW is supplied by two 200MW generators 1 & 2 with IFC are

$$\frac{dF_1}{dP_1} = 0.10 P_1 + 20 \frac{Rs}{MWH}, \quad \frac{dF_2}{dP_2} = 0.12 P_2 + 15 \frac{Rs}{MWH}$$

with power in MW and cost in Rs/H. Determine the most economic division of load between the generators saving in cost of Rs/day. There by Obtain compared equal load sharing between generators.



Sol: Given

$$\frac{dF_1}{dP_1} = 0.10 P_1 + 20 \frac{\text{Rs}}{\text{MWH}}$$

$$\frac{dF_2}{dP_2} = 0.12 P_2 + 15 \frac{\text{Rs}}{\text{MWH}}$$

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$0.1 P_1 + 20 = 0.12 P_2 + 15$$

$$0.1 P_1 - 0.12 P_2 = -5 \rightarrow \textcircled{1}$$

$$P_1 + P_2 = 300 \rightarrow \textcircled{2}$$

Solving  $\textcircled{1}$  &  $\textcircled{2}$  we get

$$P_1 = 140.90 \text{ MW}$$

$$P_2 = 159.09 \text{ MW}$$

$$P_{G1} = P_{G2} = 150 \text{ MW}$$

Cost of plant 1

$$F_1 = \int_{140.90}^{150} (0.1 P_1 + 20) dP_1$$

$$= 0.1 \left[ \frac{P_1^2}{2} \right]_{140.9}^{150} + 20 \left[ P_1 \right]_{140.9}^{150}$$

$$= \frac{0.1}{2} [(150)^2 - (140.9)^2] + 20 [150 - 140.9]$$

$$= 314.35 \text{ Rs/hr} = 314.35 \times 24 \text{ Rs/day} = 7544.4 \text{ Rs/day}$$

Cost of plant '2'

$$F_2 = - \int_{159.09}^{150} (0.12 P_2 + 15) dP_2$$

$$= - \left[ \frac{0.12}{2} [P_2^2]_{159.09}^{150} + 15 [P_2]_{159.09}^{150} \right]$$

$$= - [-168.57 + (-136.35)]$$

$$F_2 = 304.92 \text{ Rs/hr}$$

$$= 304.92 \times 24 \text{ Rs/day}$$

$$= 7318.08 \text{ Rs/day}$$

$$\text{Saving in Cost} = F_1 - F_2$$

$$= 7544.4 - 7318.08$$

$$= 226.32 \text{ Rs/day}$$

2) The fuel input of two plants are given by

$$F_1 = 0.015 P_1^2 + 16 P_1 + 15 ; F_2 = 0.025 P_2^2 + 12 P_2 + 30$$

The loss coefficients are  $B_{11} = 0.005$ ,  $B_{12} = -0.0012$ ,

$B_{22} = 0.002$ . The load to be met is 200 MW.

Determine Economic operating Schedule and corresponding cost of generation if transmission losses are negligible.

Sol: Given that

$$F_1 = 0.015 P_1^2 + 16 P_1 + 15$$

$$F_2 = 0.025 P_2^2 + 12 P_2 + 30$$

$$\frac{dF_1}{dP_1} = 0.03 P_1 + 16$$

$$\frac{dF_2}{dP_2} = 0.05 P_2 + 12$$

For economic load scheduling

$$\therefore \frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$0.03 P_1 + 16 = 0.05 P_2 + 12$$

$$0.03 P_1 - 0.05 P_2 = -4 \longrightarrow \textcircled{1}$$

$$P_1 + P_2 = 200 \longrightarrow \textcircled{2}$$

Solve eq<sup>n</sup> ① & eq<sup>n</sup> ②

$$P_1 = 75 \text{ MW}$$

$$P_2 = 125 \text{ MW}$$

$$F_1 = 0.015 (75)^2 + 16 (75) + 15$$

$$= 1299.375 \text{ Rs/hh}$$

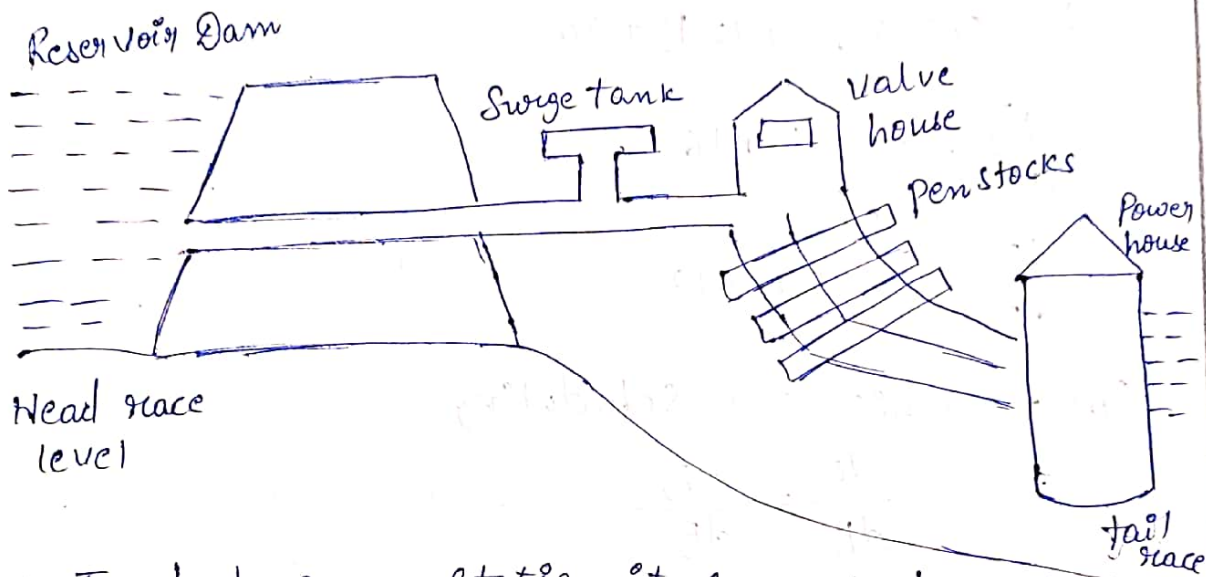
$$F_2 = 0.025 P_2^2 + 12 P_2 + 30$$

$$= 0.025 (125)^2 + 12 (125) + 30$$

$$= 1920.625 \text{ Rs/hh}$$



## Hydro Electric plant Model:



\* In hydropower station it converts hydraulic energy into electrical energy.

\* There are 3 types of hydro power plants

- 1) Run off river type.
- 2) Pumped storage type.
- 3) River type.

\* There are 3 types of head levels

- 1) Low head (10ft - 60ft) Kaplan turbine is used.
- 2) Medium head (60ft - 1000ft) Francis turbine is used.
- 3) High head ( $>1000$ ft) Pelton turbine is used.

\* An artificial storage reservoir is formed by constructing a dam across the river and a pressure tunnel is taken from reservoir to the valve house.



- \* To the valve house contains main valve for controlling water flow to the power station, and automatic isolation valve is provided for cutting off the water supply during emergency.
- \* A surge tank is provided just before the valve house for better regulation of water pressure in the system.
- \* From reservoir water is carried out to valve house through pressure tunnel and from valve house to water turbine through pipes of large diameter made of steel reinforced concrete called penstock.
- \* Water turbine converts hydraulic energy into mechanical energy and fed to alternator which converts mechanical energy into electrical energy.
- \* Water after doing useful work is discharged into tail race.
- \* Hydro plants can handle a fast change in loads & can be started easily while thermal plants takes several hours to start up and it is slow in response.
- \* In hydro thermal system a part of base load is shared by run-off river type hydroplant & remaining load is shared by coordination of both thermal & reservoir type.

Q) A system consisting of 2 plants connected by tie-line and load is located at plant 2 when 100 MW is transmitted by plant 1 a loss of 10 MW takes place on the tie line. Determine the generation scheduling of both the plants when power received  $\lambda = 25 \text{ Rs/MWh}$  IFC are

$$\frac{dC_1}{dP_1} = 0.03 P_1 + 17 \frac{\text{Rs}}{\text{MWh}} ; \frac{dC_2}{dP_2} = 0.06 P_2 + 19 \frac{\text{Rs}}{\text{MWh}}$$

Sol: Given Data

$$\frac{dC_1}{dP_1} = 0.03 P_1 + 17 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_2} = 0.06 P_2 + 19 \text{ Rs/MWh}$$

$$P_2 = P_1^2 \cdot B_{11}$$

$$P_2 = 10 \text{ MW}$$

$$P_1 = 100 \text{ MW}$$

$$10 = (100)^2 B_{11}$$

$$B_{11} = 0.001$$

$$B_{21} = B_{22} = B_{12} = 0 \quad \left\{ \because \text{load is connected to} \right. \\ \left. \text{Plant-2} \right\}$$

$$\frac{dC_1}{dP_1} = 0.03 P_1 + 17$$

$$\lambda = 25 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_2} = 0.06 P_2 + 19$$

$$L_1 = \frac{1}{1 - \frac{\partial L}{\partial P_1}}$$

$$P_2 = 0.001 P_1^2$$

$$\frac{\partial P_2}{\partial P_1} = 0.002 P_1$$

$$\frac{\partial P_2}{\partial P_2} = 0$$

$$L_1 = \frac{1}{1 - 0.002 P_1}$$

$$L_2 = \frac{1}{1 - 0} = 1$$

$$L_1 \frac{dF_1}{dP_1} = \lambda$$

$$\frac{0.03 P_1 + 17}{1 - 0.002 P_1} = 25$$

$$0.03 P_1 + 17 = 25 (1 - 0.002 P_1)$$

$$0.03 P_1 + 17 = 25 - 0.05 P_1$$

$$P_1 = 100 \text{ MW}$$

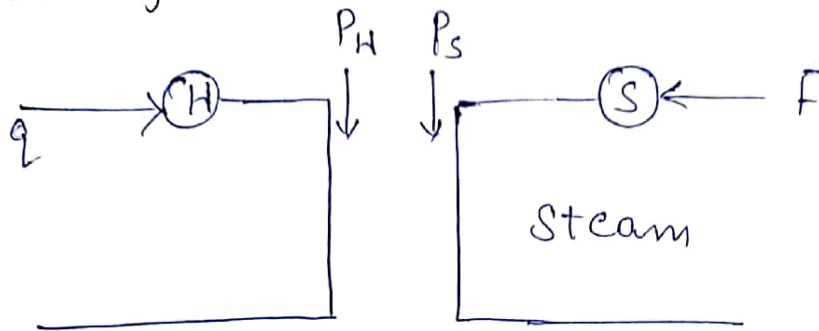
$$L_2 \frac{dF_2}{dP_2} = \lambda$$

$$0.06 P_2 + 19 = 25$$

$$\boxed{P_2 = 100 \text{ MW}}$$



## Scheduling Problems:



Two unit Hydro thermal System

In the operation of hydro electric power system three general categories of problem arises these depend upon the balance between hydrogeneration, thermal generation, load.

The economic scheduling of hydro thermal system is really a problem that scheduling water releases to satisfy all the hydrolic constraints and meet the load demand. Scheduling is developed so that cost of generation is minimized.

Consider a two unit thermal system as shown. The hydro plant has to be operated over a limited time period 'j'

$$P_{Hj}^{\max} > P_{Lj} \quad \text{for } j=1 \text{ to } j_{\max}$$

The energy available from hydro plant is insufficient to meet the load

$$\sum_{j=1}^{j_{\max}} P_{Hj} \eta_j \leq \sum_{j=1}^{j_{\max}} P_{Lj} \eta_j$$



$n_j$  = no. of hours in time period  $j$

$$\sum_{j=1}^{j_{\max}} n_j = T_{\max} = \text{Total time interval}$$

Enter amount of energy from hydro plant is taken such that cost of running steam plant is minimized.

Steam energy required

$$E_{\text{steam}} = \sum_{j=1}^{j_{\max}} P_{Lj} n_j - \sum_{j=1}^{j_{\max}} P_{Hj} n_j$$

(Steam energy) (Load) (Hydro energy)

The steam unit cannot run over all the time interval of  $T_{\max}$ . It has to run for a time period of  $N_s$ .

$$E = \sum_{j=1}^{N_s} P_{sj} n_j$$

$$E - \sum_{j=1}^{N_s} P_{sj} n_j = 0$$

$$\sum_{j=1}^{N_s} n_j < T_{\max}$$

Scheduling problem becomes

$$\text{Min } F_t = \sum_{j=1}^{N_s} F(P_{sj}) n_j$$

By using Lagrange function

$$L = \sum_{j=1}^{N_s} F(P_{sj}) n_j + d \left( E - \sum_{j=1}^{N_s} P_{sj} n_j \right)$$

$$L = F_t + \alpha \left( E - \sum_{j=1}^{N_s} P_{sj} \eta_j \right)$$

differentiating above equation w.r.t. to ' $P_{sj}$ '

$$\frac{dL}{dP_{sj}} = \frac{dF_t}{dP_{sj}} + \alpha (0 - 1) = 0$$

$$\frac{dL}{dP_{sj}} = \frac{dF_t}{dP_{sj}} = \alpha$$

$$\boxed{\frac{dF_t}{dP_{sj}} = \alpha}$$

Let  $P_s^* \rightarrow$  total

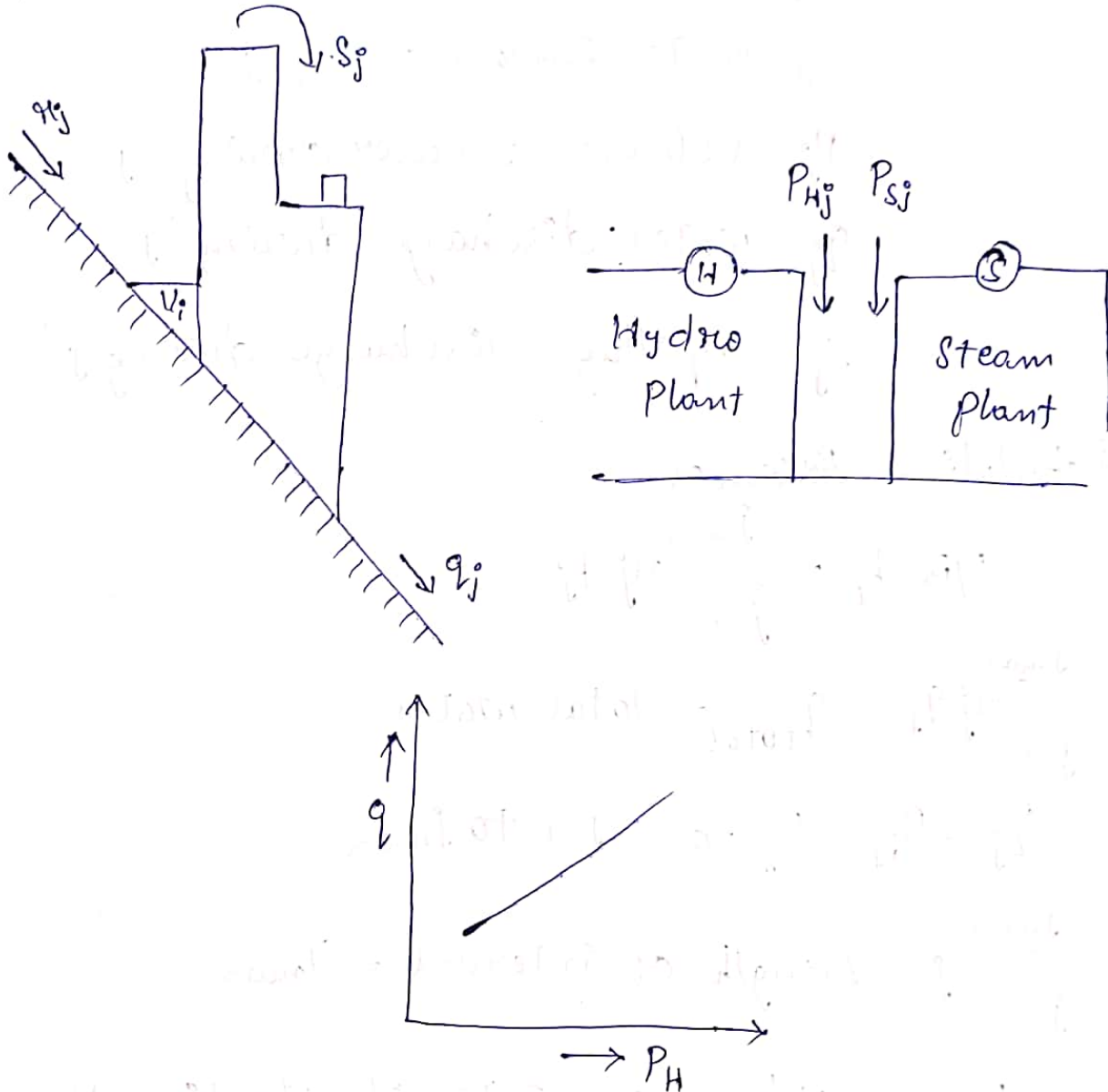
$$\begin{aligned} F_t &= \sum_{j=1}^{N_s} F(P_s^*) \eta_j \\ &= F(P_s^*) \sum_{j=1}^{N_s} \eta_j \end{aligned}$$

Cost of steam unit

$$\boxed{F_t' = F(P_s^*) T_s}$$

From the above equation Steam power plant operated at constant incremental cost. And optimum value of system generated power is  $P_s^*$  which is same in all the sub intervals.

# Short Term Hydro Thermal Scheduling:



Hydro electric unit I/p - o/p, Characteristic for Constant load

In hydro thermal Scheduling requires amount of water to be used in such a way to minimize the cost of thermal generation. Hydro plant is not sufficient to meet the load and there will be total volume of water that has to be discharged over a period of  $T_{max}$  hours.

$j$  = interval

$\eta_j$  = water inflow during 'j'

$V_j$  = Volume of water during 'j'

$q_j$  = water discharge during 'j'

$S_j$  = Spillage discharge during 'j'.

Scheduling becomes -

$$\text{Min } F_t = \sum_{j=1}^{j_{\max}} \eta_j F_j$$

$$\sum_{j=1}^{j_{\max}} \eta_j q_j = q_{\text{total}} = \text{Total water}$$

$$P_{Lj} - P_{Hj} - P_{Sj} = 0 \quad j=1 \text{ to } j_{\max}$$

$$\sum_{j=1}^{j_{\max}} \eta_j = \text{Length of interval} = T_{\max}$$

$$V_j \big|_{j=0} = \text{Volume of water at starting} = V_s$$

$$V_j \big|_{j=j_{\max}} = \text{Volume of water at ending} = V_e$$

$$q_{\min} \leq q_j \leq q_{\max}$$

$$q_j = \text{Fixed discharge}$$

Using Lagrangian's Function

$$L = \sum_{j=1}^{j_{\max}} \eta_j F(P_{Sj}) + \lambda (P_{Lj} - P_{Hj} - P_{Sj}) + \mu \left( \sum_{j=1}^{j_{\max}} \eta_j q_j - q_{\text{total}} \right) \quad \text{--- (1)}$$



Differentiating above eq<sup>n</sup> w.r.to  $P_{sj}$  and equal to zero

$$\frac{dL}{dP_{sj}} = 0$$

$$\frac{dL}{dP_{sj}} = \eta_j \frac{dF(P_{sj})}{dP_{sj}} + \lambda (-1) = 0$$

$$\eta_j \frac{dF(P_{sj})}{dP_{sj}} = \lambda$$

For specific interval  $j=k$

$$\boxed{\eta_j \frac{dF(P_{sk})}{dP_{sk}} = \lambda}$$

Differentiating eq<sup>n</sup> ① w.r.to  $P_{Hj}$  and equal to zero

$$\frac{dL}{dP_{Hj}} = 0$$

$$\frac{dL}{dP_{Hj}} = -\lambda + \eta_j \pi \frac{dQ(P_{Hj})}{dP_{Hj}} = 0$$

$$\eta_j \pi \frac{dQ(P_{Hj})}{dP_{Hj}} = \lambda$$

For a specific interval  $j=k$

$$\boxed{\eta_k \pi \frac{dQ(P_{Hk})}{dP_{Hk}} = \lambda}$$

with Losses

$$L = \sum_{j=1}^{j_{\max}} \eta_j F(P_{sj}) + \lambda (P_{Lj} - \{P_{sj} - P_{Hj} + P_{\text{loss}}\}) + \eta \sum_{j=1}^{j_{\max}} \eta_j q(P_{Hj}) + q_{\text{total}}$$

$$\frac{dL}{dP_{sj}} = 0$$

$$\frac{dL}{dP_{sj}} = \eta_j \frac{dF(P_{sj})}{dP_{sj}} + \lambda (0 - 1 - 0 + \frac{\partial P_{\text{loss}}}{\partial P_{sj}}) + 0 = 0$$

$$\eta_j \frac{dF(P_{sj})}{dP_{sj}} - \lambda + \lambda \frac{\partial P_{\text{loss}}}{\partial P_{sj}} = 0$$

$$\eta_j \frac{dF(P_{sj})}{dP_{sj}} + \lambda \frac{\partial P_{\text{loss}}}{\partial P_{sj}} = \lambda$$

For a specific interval  $j=k$

$$\boxed{\eta_k \frac{dF(P_{sk})}{dP_{sk}} + \lambda \frac{\partial P_{\text{loss}}}{\partial P_{sk}} = \lambda}$$

$$\frac{dL}{dP_{Hj}} = 0$$

$$\frac{dL}{dP_{Hj}} = 0 + \lambda \left( -1 + \frac{\partial P_{\text{loss}}}{\partial P_{Hj}} \right) + \eta_j \eta \frac{dq(P_{Hj})}{dP_{Hj}} = 0$$

$$\eta_j \eta \frac{dq(P_{Hj})}{dP_{Hj}} + \lambda \frac{\partial P_{\text{loss}}}{\partial P_{Hj}} = \lambda$$

for a specified interval  $j=k$

$$\boxed{\eta_k \eta \frac{dq(P_{Hk})}{dP_{Hk}} + \lambda \frac{\partial P_{\text{loss}}}{\partial P_{Hk}} = \lambda}$$

## Problems

1) A 2 plant system has a Steam plant near the load centre and hydroplant at remote location. The characteristics are

$$C_1 = (0.045 P_T + 26) P_T \text{ Rs/Wh}$$

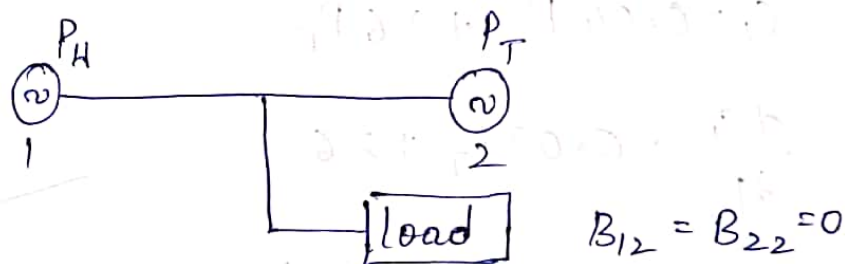
$$W_2 = (0.004 P_H + 7) P_H \text{ m}^3/\text{sec}$$

$$\eta_2 = 4 \times 10^{-4} \text{ Rs/m}^3$$

$$B_{11} = 0.0025 \text{ MW}^{-1}$$

Determine power generation at each station and power received by the load  $\lambda = 65 \text{ Rs/m}^3$

Sol:



Given that

$$C_1 = (0.045 P_T + 26) P_T \text{ Rs/Wh}$$

water  $W_2 = (0.004 P_H + 7) P_H \text{ m}^3/\text{sec}$

$$\eta_2 = 4 \times 10^{-4} \text{ Rs/m}^3$$

Loss coefficient  $B_{11} = 0.0025 \text{ MW}^{-1}$

$$\lambda = 65 \text{ Rs/m}^3$$

Co-ordination equations of thermal plant



$$\frac{dC_1}{dP_T} + \lambda \frac{\partial P_{loss}}{\partial P_T} = \lambda$$

$$P_{loss} = P_H^2 B_{11} + P_T^2 B_{22} + 2 P_T P_H B_{12}$$

$$P_{loss} = P_H^2 B_{11}$$

$$\frac{\partial P_{loss}}{\partial P_H} = 2 P_H B_{11} = 2 P_H \times 0.0025$$

$$= 0.005 P_H$$

$$\frac{\partial P_{loss}}{\partial P_T} = 0$$

$$C_1 = 0.04 P_T^2 + 26 P_T$$

$$\frac{dC_1}{dP_T} = 0.08 P_T + 26$$

$$0.08 P_T + 26 + \lambda(0) = \lambda$$

$$0.08 P_T + 26 = 65$$

$$0.08 P_T = 39$$

$$P_T = 487.5 \text{ MW}$$

Co-ordination equation for hydro generation

$$\eta_2 \frac{dW_2}{dP_H} L_2 = \lambda$$

$$W_2 = (0.004 P_H + 7) P_H$$



$$= 0.004 P_H^2 + 7 P_H$$

$$\frac{dW_2}{dP_H} = 0.004 \times 2 P_H + 7$$

$$= 0.008 P_H + 7$$

$$L_2 = \frac{1}{1 - \frac{\partial \text{loss}}{\partial P_H}}$$

$$L_2 = \frac{1}{1 - 0.005 P_H}$$

$$4 \times 10^{-4} (0.008 P_H + 7) \left( \frac{1}{1 - 0.005 P_H} \right) = 65$$

$$\frac{0.008 P_H + 7}{1 - 0.005 P_H} = 162500$$

$$0.008 P_H + 7 = 162500 (1 - 0.005 P_H)$$

$$0.008 P_H + 7 = 162500 - 812.5 P_H$$

$$0.008 P_H + 812.5 P_H = 162500 - 7$$

$$812.508 P_H = 162493$$

$$P_H = 199.99 \text{ MW}$$

$$P_{\text{loss}} = P_H^2 B_{11} = (199.9)^2 \times 0.0025$$

$$P_{\text{loss}} = 99.9 \text{ MW}$$

$$P_D = P_T + P_H - P_{\text{loss}} = 533.33 \text{ MW}$$

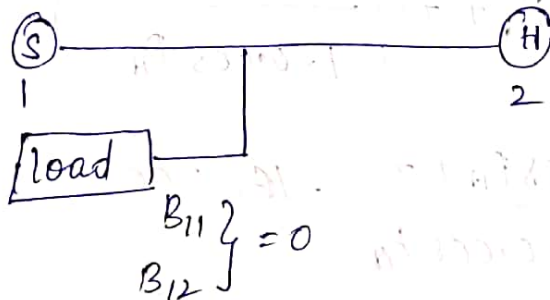
2) A two plant system having a steam plant nearer to load centre and hydroplant is located remote location, the load is 4500 MW for 16 hours a day the characteristics of units are

$$C_1 = 0.075 P_T^2 + 45 P_T + 120$$

$$W_2 = 0.002 P_H^2 + 0.6 P_H, \quad B_{22} = 0.001 \text{ MW}^{-1}$$

Find the generation schedule and daily water used by plant and daily operating cost of thermal plant for  $\eta_g = 85.5 \text{ Rs/m}^3 \text{ hr}$ .

Sol:



load demand = 4500 MW for 16 hrs day

$$C_1 = 0.075 P_T^2 + 45 P_T + 120$$

$$\frac{dC_1}{dP_T} = 0.075 \times 2 P_T + 45$$

$$= 0.15 P_T + 45$$

$$W_2 = 0.0028 P_H^2 + 0.6 P_H$$

$$\frac{dW_2}{dP_H} = 0.0028 \times 2 P_H + 0.6$$

$$= 0.0056 P_H + 0.6$$

$$B_{22} = 0.001 \text{ MW}^{-1}$$

$$P_L = P_T^2 B_{11} + P_H^2 B_{22} + 2 P_T P_H B_{12}$$

$$P_L = P_H^2 B_{22} \quad (\because B_{11} = B_{12} = 0)$$

$$\begin{aligned} \frac{\partial P_L}{\partial P_H} &= 2 P_H B_{22} = 2 P_H \times 0.001 \\ &= 0.002 P_H \end{aligned}$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_H}} = \frac{1}{1 - 0.002 P_H}$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_T}}$$

$$L_1 = \frac{1}{1 - 0}$$

$$L_1 = 1$$

Co-ordination equation for Thermal plant

$$\frac{dC_1}{dP_T} + \lambda \frac{\partial P_L}{\partial P_T} = \lambda$$

$$\frac{dC_1}{dP_T} = \lambda \left( 1 - \frac{\partial P_L}{\partial P_T} \right)$$

$$L_1 \frac{dC_1}{dP_T} = \lambda \longrightarrow \textcircled{1} \quad \left( \because L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_T}} \right)$$

Co-ordination equation for hydro plant

$$g_2 \frac{dW_2}{dP_H} + \lambda \frac{\partial P_L}{\partial P_H} = \lambda$$

$$H_2 \frac{dW_2}{dP_H} = \lambda \left( 1 - \frac{\partial P_L}{\partial P_H} \right)$$

$$L_2 H_2 \frac{dW_2}{dP_H} = \lambda \rightarrow \textcircled{2} \quad \left( \because L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_H}} \right)$$

Equating eq<sup>n</sup> ① & ②

$$L_1 \frac{dG}{dP_T} = L_2 H_2 \frac{dW_2}{dP_H}$$

$$1 (0.15 P_T + 45) = \frac{1}{1 - 0.002 P_H} 85.5 (0.0056 P_H + 0.6)$$

$$(0.15 P_T + 45) (1 - 0.002 P_H) = 85.5 (0.0056 P_H + 0.6)$$

$$0.15 P_T - 0.0003 P_T P_H + 45 - 0.09 P_H = 0.4788 P_H + 51.3$$

$$0.15 P_T - 0.0003 P_T P_H = 0.5688 P_H + 6.3$$

$$0.15 P_T - 0.5688 P_H - 0.0003 P_T P_H - 6.3 = 0 \rightarrow \textcircled{3}$$

We know that

$$P_D = P_T + P_H - P_L$$

$$4500 = P_T + P_H - P_H^2 B_{22}$$

$$4500 = P_T + P_H - P_H^2 0.001$$

$$P_T = 4500 - P_H + P_H^2 0.001 \rightarrow \textcircled{4}$$

Now substitute eq<sup>n</sup> ④ in eq<sup>n</sup> ③

$$0.15 (4500 - P_H + P_H^2 0.001) - 0.5688 P_H - 0.0003$$

$$0.0003 (4500 - P_H + P_H^2 0.001) P_H - 6.3 = 0$$



$$\Rightarrow 675 - 0.15 P_H + 0.00015 P_H^2 - 0.5688 P_H - (1.35 - 0.0003 P_H + 0.0000003 P_H^2) P_H - 6.3 = 0$$

$$\Rightarrow 675 - 0.15 P_H + 0.00015 P_H^2 - 0.5688 P_H - 1.35 P_H + 0.0003 P_H^2 - 0.0000003 P_H^3 - 6.3 = 0$$

$$668.7 - 2.0688 P_H + 0.00045 P_H^2 - 0.0000003 P_H^3 = 0$$

$$0.0000003 P_H^3 - 0.00045 P_H^2 + 2.0688 P_H - 668.7 = 0$$

$$\boxed{P_H = 342.966 \text{ MW}}$$

Now substitute  $P_H$  in eqn (4)

$$P_T = 4500 - 342.966 + (342.966)^2 \cdot 0.001$$

$$\boxed{P_T = 4274.65 \text{ MW}}$$

Daily water used by hydro plant

$$W_2 = 0.0028 (342.966)^2 + (0.6 \times 342.966)$$

$$= 535.13 \text{ m}^3/\text{sec}$$

$$= 535.13 \times 3600 \times 16 \text{ m}^3$$

$$= 30823488 \text{ m}^3$$

$$= 30.82 \times 10^6 \text{ m}^3$$

Operating cost of Thermal plant

$$C_1 = [0.075 (4274.65)^2 + 45 (4274.65) + 120] \times 16$$

$$= [1562926.697] \times 16$$

$$= 25.00 \times 10^6 \text{ RS per day}$$

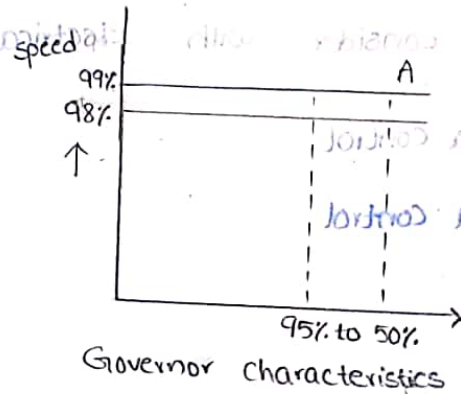
$\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$   
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$\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$   
 $\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$   
 $\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$   
 $\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$   
 $\frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} m v^2 \right) = \frac{1}{2} m v \frac{dv}{dt}$

Introduction:

The main aim of load frequency is maintained as a constant value.

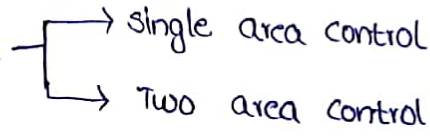


Governor characteristics

- The need for satisfactory operation of power station running in parallel, the relation between systems frequency and speed, Load depends upon Governor characteristics.
- The above diagram shows the Governor characteristics.
- The relation between load carried by the turbine speed as shown.
- If the load carried by the turbine is 25%, the speed is about 99% and if the load is increased to 50% and speed is decreased to 98%.
- In order to keep speed as a constant value, the governor is adjusted so that spring tension. In the flyball of the Governor is change.

Control area (or) Coherent area:

All the generators in such a group contribute to the coherent area so that all the generators speed up and slow down with respect to power angles the boundary of control area must consider with electrical Board company.

Control area 

Single area control:

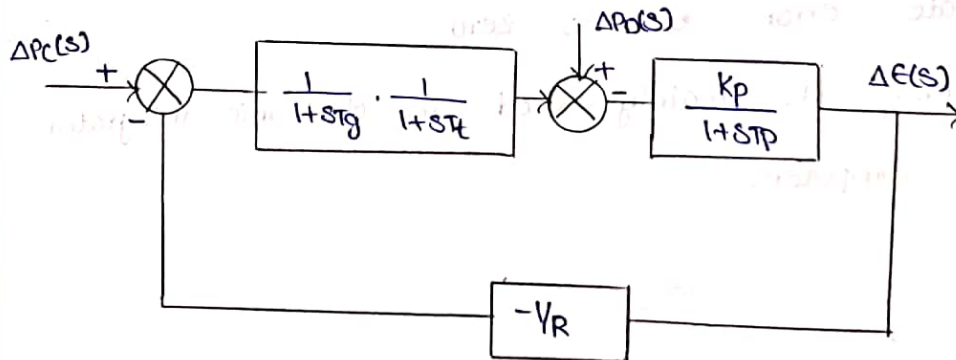
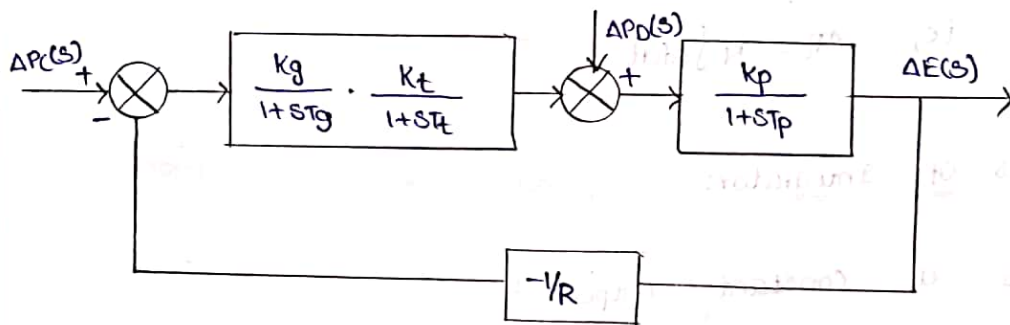
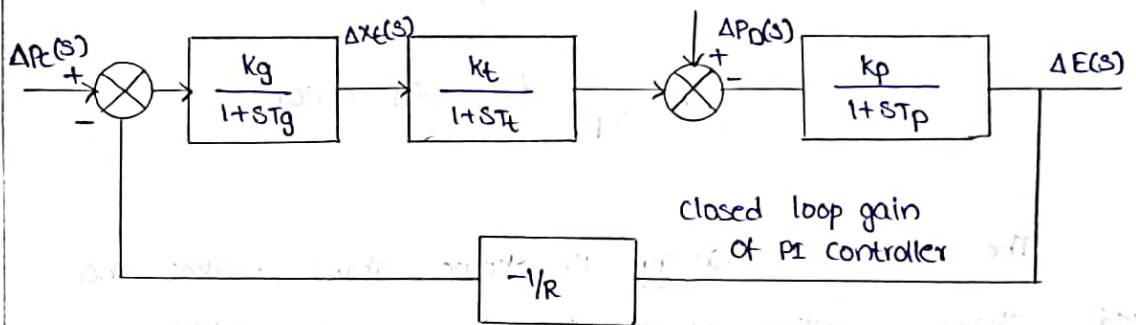
- A single area is a coherent area in which all the generators 'unison' step load changes speed settings for their static and dynamic response.
- The frequency is assumed to be constant single area is nothing but isolated power system consists of generators, turbine speed Governor and the load.



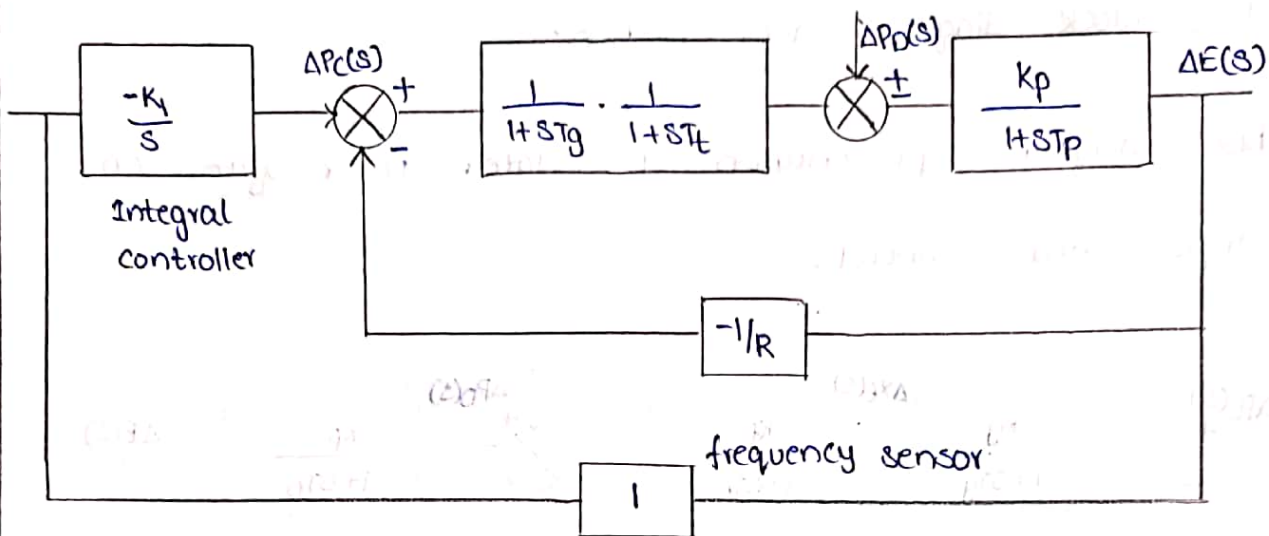
Proportional plus integral control of single area and

its block diagram representation:

Block diagram representation of isolated power system on  
single area control:



### Adding Integrator:



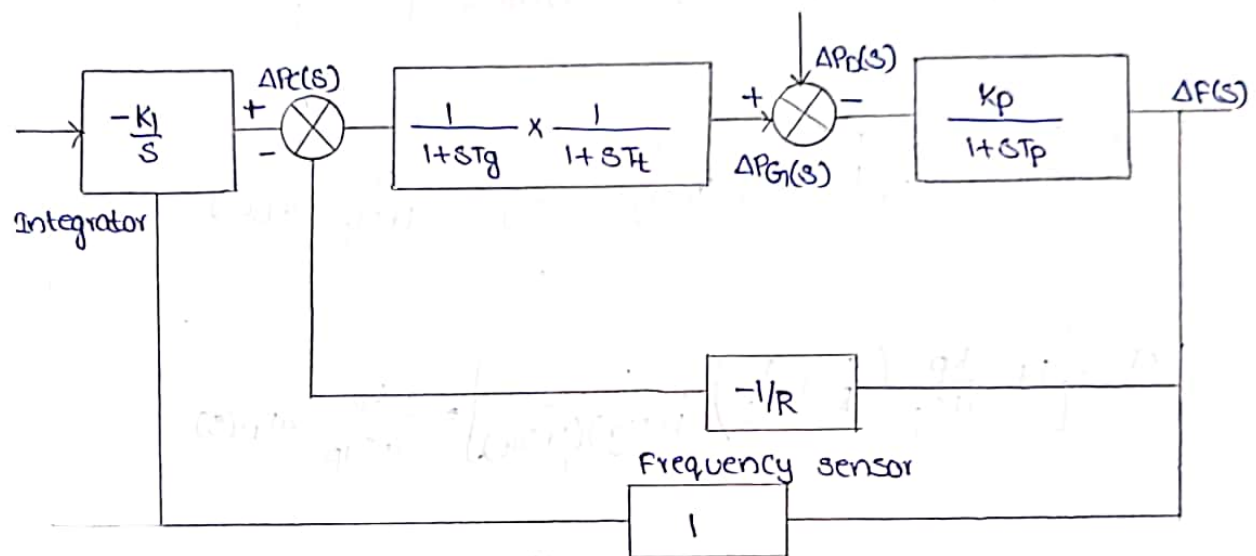
The control strategy is shown above makes the speed changes setting commanded signal when which is first amplified and then integrated.

i.e,  $\Delta P_c = -K_I \int \Delta E dt$

### Advantages of Integrator:

- Maintains a constant output.
- Speed changer position attains a constant value.
- Steady state error becomes zero.
- The integration is mainly used in electronic integrator to analog computers.

Steady state response:



$$\Delta P_C = -K_I \int \Delta F dt$$

Applying laplace transform

$$\Delta P_C(s) = -\frac{K_I}{s} \Delta F(s)$$

From the block diagram,

$$\left[ -\frac{1}{R} \Delta F(s) - \frac{K_I}{s} \Delta F(s) \right] \frac{1}{(1+sT_g)(1+sT_t)} = \Delta P_G(s)$$

$$-\Delta F(s) \left[ \frac{1}{R} + \frac{K_I}{s} \right] \frac{1}{(1+sT_g)(1+sT_t)} = \Delta P_G(s) \rightarrow \textcircled{1}$$

$$\Delta F(s) = \frac{K_p}{1+sT_p} [\Delta P_G(s) - \Delta P_D(s)] \rightarrow \textcircled{2}$$

Substitute eq① in eq②

$$\Delta F(s) = \frac{K_p}{1+sT_p} \Delta P_G(s) - \frac{K_p}{1+sT_p} \Delta P_D(s)$$

$$\Delta F(s) = \frac{k_p}{1+sT_p} \left[ -\Delta F(s) \left( \frac{1}{R} + \frac{k_I}{s} \right) \frac{1}{(1+sT_g)(1+sT_t)} \right] - \frac{k_p}{1+sT_p} \Delta P_D(s)$$

$$\Delta F(s) + \frac{k_p}{1+sT_p} \Delta F(s) \left( \frac{1}{R} + \frac{k_I}{s} \right) \frac{1}{(1+sT_g)(1+sT_t)} = -\frac{k_p}{1+sT_p} \Delta P_D(s)$$

$$\Delta F(s) \left[ 1 + \frac{k_p}{1+sT_p} \left( \frac{1}{R} + \frac{k_I}{s} \right) \frac{1}{(1+sT_g)(1+sT_t)} \right] = -\frac{k_p}{1+sT_p} \Delta P_D(s)$$

$$\Delta F(s) = \frac{-\frac{k_p}{1+sT_p} \Delta P_D(s)}{1 + \frac{k_p}{1+sT_p} \left( \frac{1}{R} + \frac{k_I}{s} \right) \frac{1}{(1+sT_g)(1+sT_t)}}$$

$$\Delta P_D(s) = \frac{\Delta P_D}{s}$$

$$\Delta F(s) = \frac{-\frac{k_p}{1+sT_p} \frac{\Delta P_D}{s}}{(1+sT_g)(1+sT_p)(1+sT_t) + \frac{k_p}{R} + \frac{k_p k_I}{s}}$$

$$= \frac{-\frac{k_p}{1+sT_p} \frac{\Delta P_D}{s} (1+sT_p)(1+sT_g)(1+sT_t)}{(1+sT_g)(1+sT_p)(1+sT_t) + \frac{k_p}{R} + \frac{k_p k_I}{s}}$$

$$= \frac{-k_p \frac{\Delta P_D}{s} (1+sT_g)(1+sT_t)}{(1+sT_g)(1+sT_p)(1+sT_t) + \frac{k_p}{R} + \frac{k_p k_I}{s}}$$



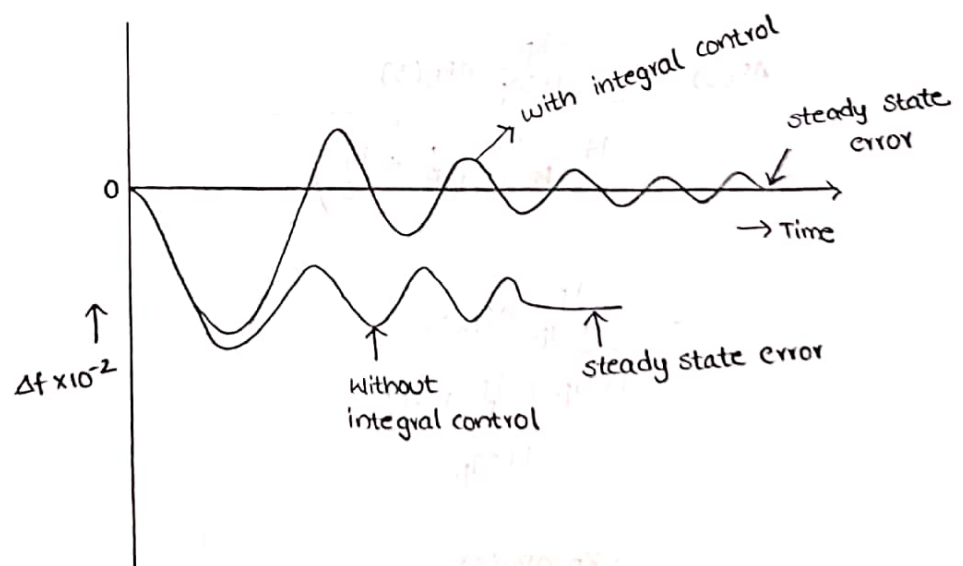
$$\Delta F(s) = \frac{-K_p \frac{\Delta P_D}{s} (1+sT_g)(1+sT_t)}{R s (1+sT_p)(1+sT_g)(1+sT_t) + K_p s + K_p K_I R}$$

$$= \frac{-K_p \Delta P_D (1+sT_g)(1+sT_t) R}{R s (1+sT_p)(1+sT_g)(1+sT_t) + K_p s + K_p K_I R}$$

Apply final value theorem to above equation

$$\Delta F(t) = \Delta F|_{\text{steady state}} = \lim_{s \rightarrow 0} s \Delta F(s) = 0$$

Dynamic response:



### Assumptions:

- Area controlled error is a continuous signal.
- $T_g$  and  $T_t$  are neglected.
- Non-linearities are neglected.
- Generator can generate according to speed changes setting.
- Speed changes setting is a electro mechanical device and its response must be instantaneous.

$$\Delta F(s) = \frac{\frac{-K_p}{1+sT_p} \Delta P_D(s)}{1 + \frac{K_p}{1+sT_p} \left( \frac{1}{R} + \frac{K_I}{s} \right) \frac{1}{(1+sT_g)(1+sT_t)}}$$

$T_g$  &  $T_t$  are neglected

$$\Delta F(s) = \frac{\frac{-K_p}{1+sT_p} \Delta P_D(s)}{1 + \frac{K_p}{1+sT_p} \left( \frac{1}{R} + \frac{K_I}{s} \right)}$$

$$= \frac{\frac{-K_p}{1+sT_p} \Delta P_D(s)}{\frac{1+sT_p + \frac{K_p}{R} + \frac{K_p K_I}{s}}{1+sT_p}}$$

$$\Delta F(s) = \frac{-K_p \Delta P_D(s)}{1+sT_p + \frac{K_p}{R} + \frac{K_p K_I}{s}}$$

$$\Delta P_D(s) = \frac{\Delta P_D}{s}$$

$$\Delta F(s) = \frac{-K_p \frac{\Delta P_D}{s}}{1 + sT_p + \frac{K_p}{R} + \frac{K_p K_I}{s}}$$

Multiply numerator and denominator with 's'

$$\Delta F(s) = \frac{-K_p \frac{\Delta P_D}{s} \times s}{s + s^2 T_p + \frac{K_p s}{R} + \frac{K_p K_I}{s} s}$$

$$\Delta F(s) = \frac{-K_p \Delta P_D}{T_p \left( \frac{s}{T_p} + s^2 + \frac{K_p s}{T_p R} + \frac{K_p K_I}{T_p} \right)}$$

$$\Delta F(s) = \frac{-K_p}{T_p} \frac{\Delta P_D}{s^2 + \frac{s}{T_p} \left( 1 + \frac{K_p}{R} \right) + \frac{K_p K_I}{T_p}}$$

Consider characteristic equation

$$s^2 + \frac{s}{T_p} \left( 1 + \frac{K_p}{R} \right) + \frac{K_p K_I}{T_p} = 0$$

$$(s + \alpha)^2 + \omega^2 = 0$$

$$\left[ s + \left( \frac{1 + \frac{K_p}{R}}{2T_p} \right) \right]^2 + \left[ \frac{K_p K_I}{T_p} - \left( \frac{1 + \frac{K_p}{R}}{2T_p} \right)^2 \right] = 0$$

$$\alpha = \frac{1 + \frac{K_p}{R}}{2T_p}, \quad \omega = \frac{K_p K_I}{T_p} - \left( \frac{1 + \frac{K_p}{R}}{2T_p} \right)^2$$

Case-i:

$$\omega^2 = 0$$

$$\left( \frac{K_p K_I}{T_p} - \left( \frac{1 + \frac{K_p}{R}}{2T_p} \right)^2 \right) = 0$$

$$\frac{K_p K_I}{T_p} = \left( \frac{1 + \frac{K_p}{R}}{2T_p} \right)^2$$

$$K_I = \frac{\left( 1 + \frac{K_p}{R} \right)^2}{4K_p T_p}$$

$$K_I = K_I (\text{critical})$$

Response  $\rightarrow$  critically damped

$$\text{Roots} \rightarrow e^{-\alpha t}, te^{-\alpha t}$$

Case-ii:

$$\omega^2 > 0$$

$$(s + \alpha)^2 + \omega^2 > 0$$

$$(s + \alpha)^2 = -\omega^2$$

$$s + \alpha = -j\omega$$

$$s = -\alpha \pm j\omega$$

response  $\rightarrow$  over damped and oscillatory, roots  $\rightarrow e^{-\alpha t} \cos \omega t, e^{-\alpha t} \sin \omega t$

$$K_I > K_I (\text{critical})$$

Case-iii:

$$\omega^2 < 0$$

$$(s + \alpha)^2 + \omega^2 < 0$$

$$s + \alpha = -\omega = -\beta$$

$$s = -\alpha \pm j\beta$$

Response  $\rightarrow$  damped, roots  $\rightarrow e^{-\beta t}, e^{-\beta t}$

$$K_I < K_I (\text{critical})$$

In all the cases, by applying final value theorem,  $AF(t)$  becomes zero but it is observed that transient frequency error becomes a finite value.



A 200MVA synchronous generator is operated at 3000rpm, 50Hz, a load of 40MW is suddenly applied to the machine and the station valve turbine opens only after 0.4sec due to time lag in generator action. Calculate the frequency to which generator voltage drops before the steam flow commences to increase and to meet the new load.

$H = 5.5 \text{ KW-sec/KVA}$  of Generator.

Sol:

Given,

rating of generator = 200MVA

$N = 3000 \text{ rpm}$

$f = 50 \text{ Hz}$

load applied = 40MW

Time taken to open the valve when load applied = 0.4sec

energy lost by rotor =  $40 \times 0.4$

= 16 MW-sec

energy stored  $H = 5.5 \text{ KW-sec/KVA}$

$$= \frac{5.5 \times 1000}{1000} \frac{\text{KW-sec}}{\text{KVA}}$$

$$= 5500 \text{ KW-sec/MVA}$$

$$= 5500 \text{ KW-sec} \times 200$$

$$= 1.1 \times 10^6 \text{ KW-sec}$$

$$H = 1100 \text{ MW-sec}$$

$$\text{New frequency} = \sqrt{\frac{1100-16}{1100}} \times 50$$

$$= 49.63 \text{ Hz}$$

- A single area consists of two generators if the following parameters.

Generator-1 : 1200 MVA,  $R=6\%$  on machine base

Generator-2 : 1000 MVA,  $R=4\%$  on machine base.

The units of sharing 18MW at normal frequency 50Hz. Unit-1 supplies 1000MW, Unit-2 supplies 800MW. The load now increased by 200MW. Find steady state frequency and generation of each unit if  $B=0$  (pu MW/Hz), steady state frequency and generation of each unit if  $B=1.5$

Sol:

Given,

$$P_1 = 1000 \text{ MW}$$

$$P_2 = 800 \text{ MW}$$

Generator-1 : 1200 MVA,  $R=6\%$

Generator-2 : 1000 MVA,  $R=4\%$

$$f = 50 \text{ Hz}$$

Increase in load = 200 MW

$$\text{change in frequency } \Delta f = \frac{-\Delta P}{\frac{1}{R_1} + \frac{1}{R_2} + B}$$

Common base = 1000 MVA

$$\Delta P = \frac{200}{1000} = 0.2$$

$$R_1 = 1000 \times \frac{0.06}{1200} = 0.05$$

$$R_2 = 1000 \times \frac{0.04}{1000} = 0.04$$

i) steady state frequency  $B=0$

$$\Delta f = \frac{-0.2}{\frac{1}{0.05} + \frac{1}{0.04} + 0}$$

$$= -4.44 \times 10^{-3}$$

steady state frequency deviation =  $f + \Delta f$

$$= 50 - 4.44 \times 10^{-3}$$

$$= 49.99 \text{ Hz}$$

change in generation

$$\Delta P_1 = \frac{-\Delta f}{R_1} = \frac{-(-4.44 \times 10^{-3})}{0.05} = 0.088 \text{ PU}$$

$$\Delta P_1 = 0.088 \times 1000 = 88 \text{ MVA}$$

$$\Delta P_2 = \frac{-\Delta f}{R_2} = \frac{-(-4.44 \times 10^{-3})}{0.04} = 0.11 \text{ PU}$$

$$\Delta P_2 = 0.11 \times 1000 = 110 \text{ MVA}$$

ii) steady state frequency  $B=1.5$

$$\Delta F = \frac{-0.2}{\frac{1}{0.05} + \frac{1}{0.04} + 1.5}$$

$$\Delta F = -4.3 \times 10^{-3}$$

$$\begin{aligned}\text{Steady state frequency deviation} &= f + \Delta f \\ &= 50 - 4.3 \times 10^{-3} \\ &= 49.99 \text{ Hz}\end{aligned}$$

Change in generation

$$\Delta P_1 = \frac{-(-4.3 \times 10^{-3})}{0.05} = 0.086 \text{ PU}$$

$$\Delta P_1 = 0.086 \times 1000 = 86 \text{ MVA}$$

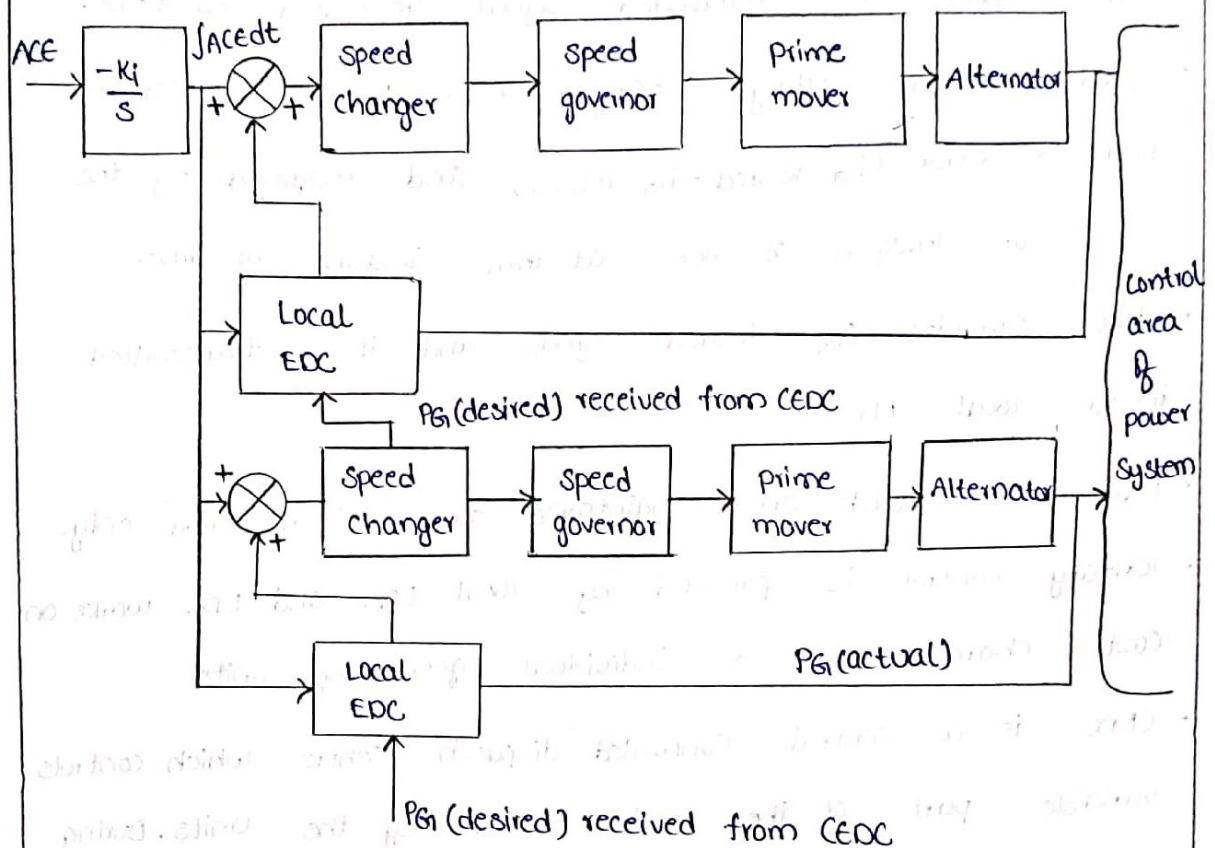
$$\Delta P_2 = \frac{-\Delta F}{R_2} = \frac{-(-4.3 \times 10^{-3})}{0.04} = 0.107 \text{ PU}$$

$$\Delta P_2 = 0.107 \times 1000 = 107 \text{ MVA}$$

//



## Load frequency and Economic dispatch control:



- LFC and EDC place a vital role in modern power system analysis.
- In LFC zero steady state error and a fast dynamic response is achieved by integral control action but this control is independent of EDC. Since there is no control over economic loading of various generating units of the controlling area.
- Some control over loading of individual units can be done by adjusting the gain factor  $K_i$  of the integral signal of ACE fed to the each generating unit but it is not the satisfactory control. Hence an independent control of LFC and EDC must be needed.

- LFC is a fast acting control and EDC is a slow acting control in which speed changer setting varies with respect to commanded signal generated by CEDC.
- Speed changer setting changes with respect to economic dispatch error ( $P_{G, \text{desired}} - P_{G, \text{actual}}$ ) and is modified by the signal of integral of ACE at that instant of time.
- CEDC provides  $P_{G, \text{desired}}$  signal and it is transmitted to local EDC.
- Economic dispatch error maintains for a short period only.
- Tertiary control is provided by local EDC and EDC works on cost characteristics of individual generating units.
- CEDC is a central controlled dispatch centre which controls variable part of the load carried by the units. During peak load hours medium size fossil fuels, hydro units, diesel units, gas turbines are used.
- CEDC monitors area of frequency, output of units, constant power flow, constant frequency, tie line power flow to interconnected area used for ALFC (Area load frequency control).
- Rising and lowering of power signal is fed to the turbine governor.
- Hence LFC is coordinated with EDC for maintaining economic loading of units and satisfying LFC objectives.



## UNIT-VI

### Reactive Power Control

#### Introduction:

- \* Reactive Power is defined as the ratio of Active Power to the apparent power.
- \* It plays a key role in AC transmission system for making the system voltage as stable.
- \* Frequency is maintained as a constant value when there is a balance between active and reactive power.

#### Generation & Absorption of Reactive Power

The following are the elements.

##### 1) Alternators:

An over excited alternator generates reactive power and an under excited alternator absorbs reactive power. In general alternator operated in over excited mode acts as a main source of reactive power.

The capability of supplying reactive power depends on short circuit ratio.

##### 2) Overhead Lines:

An overhead line when fully loaded absorbs reactive power given by  $I^2 X$

Where  $I \rightarrow$  Line Current

$X \rightarrow$  Line Reactance in  $\Omega/\text{ph}$ .

when it is lightly loaded it generates reactive power given by  $V^2 \omega c$

where  $V \rightarrow$  Phase voltage

$\omega \rightarrow$  System frequency in radians.

$c \rightarrow$  Line to earth capacitance

### 3) Transformers:

Transformers always absorb reactive power and given by  $I^2 \frac{V}{I_{\text{rated}}} X_t \text{ VAR/ph}$

where  $I \rightarrow$  Phase current

$V \rightarrow$  Phase voltage

$I_{\text{rated}} \rightarrow$  Rated phase current

$X_t \rightarrow$  Per Unit transformer reactance.

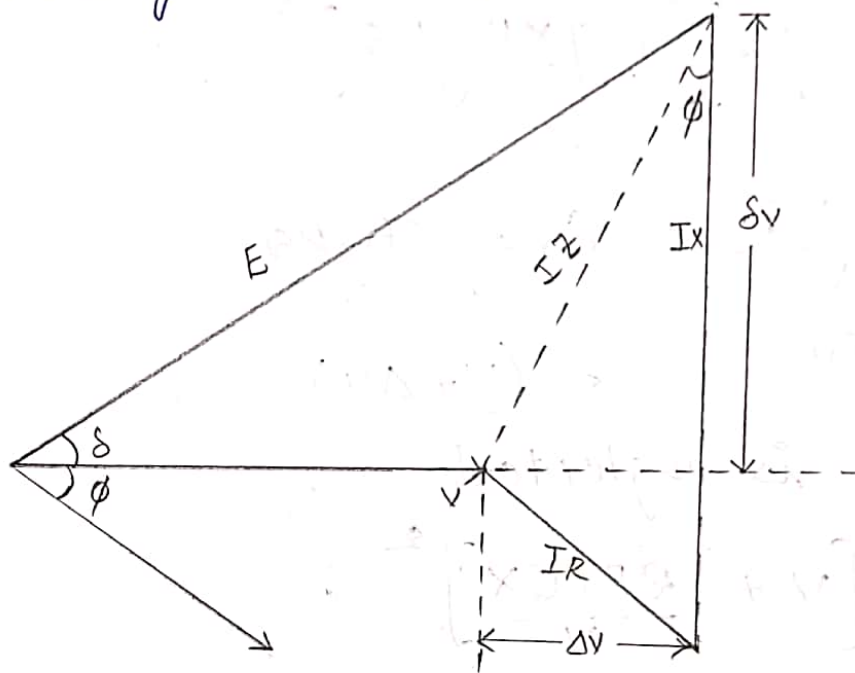
### 4) Cables:

Cables generate reactive power and it is due to variable cable capacitances.



# Role of Reactive power on voltage & Voltage Regulation

## Phasor diagram of simple transmission network



Consider a simple transmission network and there will be interrelation between sending end voltage, receiving end voltage and power angle.

From the phasor diagram

$$E^2 = (V + \Delta V)^2 + (\delta V)^2$$

$$E^2 = (V + IR \cos \phi + IX \sin \phi)^2 + (IX \cos \phi - IR \sin \phi)^2$$

$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

$$I \cos \phi = \frac{P}{V}$$

$$I \sin \phi = \frac{Q}{V}$$

$$E^2 = \left[ V + \frac{RP}{V} + \frac{XQ}{V} \right]^2 + \left[ \frac{XP}{V} - \frac{RQ}{V} \right]^2$$

$$E^2 = \left[ V + \frac{RP + XQ}{V} \right]^2 + \left[ \frac{XP - RQ}{V} \right]^2$$

$$\Delta V = \frac{RP + XQ}{V} ; \quad S_V = \frac{XP - RQ}{V}$$

but  $S_V \ll (V + \Delta V)$

So  $S_V$  is neglected

$$E^2 = \left[ V + \frac{RP + QX}{V} \right]^2$$

$$E = V + \frac{RP + QX}{V}$$

$$E - V = \frac{RP + QX}{V} = \Delta V$$

Assume Line Resistance  $R = 0$

$$E - V = \frac{XQ}{V}$$

From the above equation voltage and voltage regulation depends on reactive power.

## Over view of Reactive Power Control

- \* The economics of power transmission have to transmit as much as power for the given transmission line.
- \* Continuity of service should be maintained with security & reliability of the given transmission network depends on load centres.
- \* In new transmission network can be installed based on load centres.
- \* Development of right of way of hydro electric sources.
- \* The above three things can be done by compensation scheme to make AC transmission technically and economically strong.
- \* Reactive power need to control the voltage above a steady state value provide quality service to consumer load premises.
- \* For generating a pure sine wave it needs simple reactive power.
- \* Reactive power is important for the operation of AC current in electrical power systems.
- \* Reactive power generates harmonics.
- \* In distribution network voltage drops & increasing transmission capacity is done by



Series Capacitors by reducing series inductive reactance.

\* The impedance of the network is reactive so the transmission of active power requires a phase angle between sending end & receiving end voltage.

\* Reactive power improves

- Increasing transmission capacity.

- Better voltage regulation.

- Maintaining stability.

- Provides quality service to consumer loads.

- Prevents unnecessary flow of reactive power on the transmission line.

• Develops static type of controllers therefore practically direct measurement of reactive power is zero & power factor is less than unity.

Reactive Power Compensation in transmission system

Reactive power is closely related to voltage

Control. Apparent power  $S = P + jQ$ .

where  $S$  = Apparent power in KW

$Q$  = Reactive power in KVAR

In various equipment in the network generates or absorbs reactive power.



## Synchronous Machine:

An over excited Synchronous motor operating at no load generates a leading reactive power. It is flexible reactive power source & generates reactive power by changing excitation.

Synchronous Condenser is located nearer to the load to reduce losses in the system. Industrial lagging loads require static capacitors for generating leading reactive power.

## Advantages & Disadvantages of different types of Compensating Equipment for transmission system

Compensation Equipment	Advantages	Disadvantages
1) Switched Shunt reactor	1) Simple in principle & Construction	1) Fixed in value
2) Switched Shunt Capacitor	2) Simple in principle & Construction	2) Fixed in value Switching transients
3) Series Capacitor	3) Simple in principle Performance relatively insensitive to location.	3) Requires over voltage protection & subharmonic filters, limited overload capability.
4) Synchronous Condenser	4) Has useful overload capability low harmonics	4) High maintenance requirement, slow control response, performance, sensitivity location, requiring strong foundation.

Compensator Equipment	Advantages	Disadvantages
5) Poly phase Saturated Reactor	5) Very rugged construction, large over load capability, no effect on fault level, low harmonics	5) Essentially fixed in value, performance sensitive to location, noisy
6) Thyristor Controlled reactor [TCR]	6) Fast response, fully controllable, no effect on fault level, can be rapidly repeated	6) Generate harmonics, performance sensitive to control.
7) Thyristor Switched Capacitor [TSC]	7) Can be rapidly repaired after failure, no harmonics	7) No inherent absorbing capability to limit over voltages, complex bus network & controls, low frequency response with system, performance sensitive to location.



## Load Compensation

Compensation is basically two types

1) Line Compensation.

2) Load Compensation

The Compensation is done at the mid point of the line or uniformly distributed point on the line is called line Compensation.

The Compensation is done nearer to the load is called load Compensation.

Load Compensation is the management of reactive power to provide quality of supply to consumer loads.

Reactive power is adjusted with respect to load.

The main objective of load Compensation is

- 1) Better voltage profile
- 2) Power Factor Correction
- 3) Load balancing

### Better Voltage Profile:

The voltage profile should be within  $\pm 5\%$  voltage variation is due to imbalance of generation and absorption of reactive power in the system.

If the generated reactive power is greater than absorbed then voltage level increases and vice versa.

If the generated reactive power is equal to absorbed reactive power then a flat voltage profile is maintained.

Hence reactive power must be controlled with respect to load for maintaining a flat voltage profile.

One of the methods is to have the system of large strength i.e.; interconnecting a large sized machine to large number of lines which reduces impedance and maintains a flat voltage profile that results in high fault levels and requires high capacity switch gear equipment. So it is not economical method.

The network should be designed based on active power transfer capability and reactive power is supplied by shunt compensation.

Shunt compensation does not increase fault levels and maintains a flat voltage profile.



### Power Factor Connection:

- 1) It is economically & technically to operate the system at near unity power factor.
- 2) Power factor connection means generation of reactive power with respect to load.
- 3) The power factor connection equipment is connected at a distance and transmit to the load otherwise losses will increase.
- 4) To operate the system at unity power factor some of the utilities impart penalty factor on the loads operating at low power factor.

### Load Balancing:

- 1) Load Compensation means load balancing.
- 2) In three phase systems under un-balanced condition it generates negative sequence currents. Results in high dangerous especially to rotating machines.

### Ideal Load Compensator Functions:

- 1) It should maintain a constant voltage at all its terminals.
- 2) It should operate independently in all the three phases.

3) It should provide adjustable and controllable reactive power with respect to load.

Compensation is required for some of the loads like very large induction motor, arc furnace, arc welders, Induction furnaces, Induction welders. These loads are called non-linear loads. For example non-linear load like arc furnace generates harmonics which in turn requires filters. When synchronous motor is used as load it improves voltage profile and power factor since it is called Synchronous Capacitor.

Per Unit Regulation for series impedance

$$\text{Voltage drop } \Delta V = \frac{IR}{V} \cos \phi \pm \frac{IX}{V} \sin \phi$$

'+' sign  $\Rightarrow$  Inductive load

'-' sign  $\Rightarrow$  Capacitive load

$\phi \Rightarrow$  Power factor angle.

Consider a capacitively load

$$\frac{IR}{V} \cos \phi \cong \frac{IX}{V} \sin \phi$$

$$\boxed{\Delta V = 0}$$



A purely reactive compensator eliminates voltage variations in both active and reactive powers but it is not possible to achieve both unity power factor and zero voltage regulation.

An alternative method is used i.e short ckt capacity of the bus,  $X:R$  ratio, active and reactive power loads.

A 3- $\phi$  short circuit is occurred due to load bus.

Short circuit apparent power

$$S_{sc} = E I_{sc} = E \cdot \frac{E}{Z_{sc}} = \frac{E^2}{Z_{sc}} \rightarrow \textcircled{1}$$

Voltage drop for a series impedance.

$$\Delta V = (R + jX) \left( \frac{P - jQ}{V} \right)$$

$$\Delta V = \frac{RP - jRQ + jXP + XQ}{V}$$

$$\Delta V = \left[ \frac{RP + XQ}{V} \right] + j \left[ \frac{XP - RQ}{V} \right] \rightarrow \textcircled{2}$$

$$\Delta V = \Delta V_R + j \Delta V_X$$

$$R = |Z| \cos \phi_{sc}$$

$$X = |Z| \sin \phi_{sc}$$

$$\text{But } S_{sc} = \frac{E^2}{Z_{sc}}$$

$$Z_{sc} = \frac{E^2}{S_{s.c}}$$

$$R = \frac{E^2}{S_{s.c}} \cos \phi_{s.c}$$

$$X = \frac{E^2}{S_{s.c}} \sin \phi_{s.c}$$

Substitute 'R' & X values in eq<sup>n</sup> ②

$$\Delta V = \left[ \frac{E^2}{S_{s.c}} \cos \phi_{s.c} P + \frac{E^2}{S_{s.c}} \sin \phi_{s.c} Q \right] +$$

$$j \left[ \frac{E^2}{S_{s.c}} \sin \phi_{s.c} P - \frac{E^2}{S_{s.c}} \cos \phi_{s.c} Q \right]$$

$$\Delta V = \frac{E^2}{S_{s.c}} \left[ \frac{\cos \phi_{s.c} P + \sin \phi_{s.c} Q}{V} \right] + j \left[ \frac{\sin \phi_{s.c} P - \cos \phi_{s.c} Q}{V} \right]$$

$$= \frac{E^2}{V S_{s.c}} \left[ \cos \phi_{s.c} P + \sin \phi_{s.c} Q \right] + j \left[ \sin \phi_{s.c} P - \cos \phi_{s.c} Q \right]$$

Consider  $E \approx V$

$$= \frac{V^2}{V S_{s.c}} \left[ \cos \phi_{s.c} P + \sin \phi_{s.c} Q \right] + j \left[ \sin \phi_{s.c} P - \cos \phi_{s.c} Q \right]$$

$$= \frac{V}{S_{s.c}} \left[ \cos \phi_{s.c} P + \sin \phi_{s.c} Q \right] + j \left[ \sin \phi_{s.c} P - \cos \phi_{s.c} Q \right]$$



$$\frac{\Delta V}{V} = \frac{1}{S_{s.c}} \left[ \cos \phi_{s.c} P + \sin \phi_{s.c} Q \right] + j \left[ \sin \phi_{s.c} P - \cos \phi_{s.c} Q \right]$$

$$\frac{\Delta V}{V} = \frac{1}{S_{s.c}} \left[ \cos \phi_{s.c} P + \sin \phi_{s.c} Q \right] + j \frac{1}{S_{s.c}} \left[ \sin \phi_{s.c} P - \cos \phi_{s.c} Q \right]$$

$$\left\{ \begin{array}{l} \frac{1}{S_{s.c}} \left[ P \cos \phi_{s.c} + Q \sin \phi_{s.c} \right] = \frac{\Delta V_R}{V} \\ \frac{1}{S_{s.c}} \left[ P \sin \phi_{s.c} - Q \cos \phi_{s.c} \right] = \frac{\Delta V_X}{V} \end{array} \right\}$$

$$\frac{\Delta V}{V} = \frac{\Delta V_R}{V} + j \frac{\Delta V_X}{V}$$

$\frac{\Delta V_X}{V}$  has no effect on voltage magnitude

So it can be ignored

$$\frac{\Delta V_R}{V} = \frac{1}{S_{s.c}} \left[ P \cos \phi_{s.c} + Q \sin \phi_{s.c} \right]$$

$P$  &  $Q$  are not valid

So consider  $\Delta P$  &  $\Delta Q$  which are valid

$$\frac{\Delta V_R}{V} = \frac{1}{S_{s.c}} \left[ \Delta P \cos \phi_{s.c} + \Delta Q \sin \phi_{s.c} \right]$$

$\Delta P \rightarrow$  Active power does not effect voltage  
So it is also can be ignored

$$\frac{\Delta V_H}{V} = \frac{1}{S_{s.c}} \Delta Q \sin \phi_{s.c}$$

$$\phi_{s.c} = \tan^{-1} \frac{X}{R}$$

$$\frac{X}{R} > 4$$

$$\sin \phi_{s.c} = 1$$

$$\frac{\Delta V}{V} = \frac{\Delta Q}{S_{s.c}}$$

### Specifications of Load Compensator

The factors and parameters to be considered for specifying a load compensator.

1) Maximum continuous rating of reactive power (Q) is required for generation and absorption.

2) Overload rating and duration.

3) Rated voltage & limits of voltage b/w which reactive power rating must not be exceeded.

4) Voltage Regulation

- 5) Reliability is Required
- 6) Special Control Requirements
- 7) Frequency and its Variation
- 8) Maintenance, Spare parts for future expansion
- 9) Performance with unbalanced loads & voltages
- 10) Response time of Compensator for a Specified disturbance.

① A 3- $\phi$  50Hz 3000V motor develops 600 H.P (metric). The power factor is 0.75 lagging and the efficiency is 0.95 a bank of Capacitor is connected in delta across the supply terminals and power factor is rated upto 0.98 lagging. Each of the Capacitance unit is build of 5 similar 600V Capacitors. Determine the Capacitance of each capacitor.

Given data

$V_L = 3000V$	$\cos \phi_1 = 0.75 \text{ lag}$
$F = 50\text{Hz}$	$\phi_1 = 41.40$
$P = 600 \text{ h.p (metric)}$	$\eta = 0.95$
1 metric h.p = 746W	$\cos \phi_2 = 0.98$ { when a Capacitor bank is connected across supply }
1 British h.p = 735.5W	$\phi_2 = 11.47$
$P = 600 \times 746 = 447.6 \text{ kW}$	$V_C = 600V$



Capacitance  $C = ?$

$$P = \sqrt{3} V_L I_L \cos \phi_1$$

$$I_L = \frac{447.6 \times 10^3}{\sqrt{3} \times 3000 \times 0.75}$$

$$I_L = 114.85 \text{ A}$$

Active Component of Current

$$I_o = I_L \cos \phi_1$$

$$= 114.85 \times 0.75$$

$$= 86.14 \text{ A}$$

Capacitance Current

$$I_c = I_o (\tan \phi_1 - \tan \phi_2)$$

$$= 86.14 [\tan(41.4^\circ) - \tan(11.4^\circ)]$$

$$= 58.46 \text{ A}$$

$$I_c = \frac{V}{X_c}$$

$$X_c = \frac{V}{I_c} = \frac{600}{58.46}$$

$$X_c = 10.26 \Omega$$

$$\frac{1}{2\pi f C} = 10.26$$

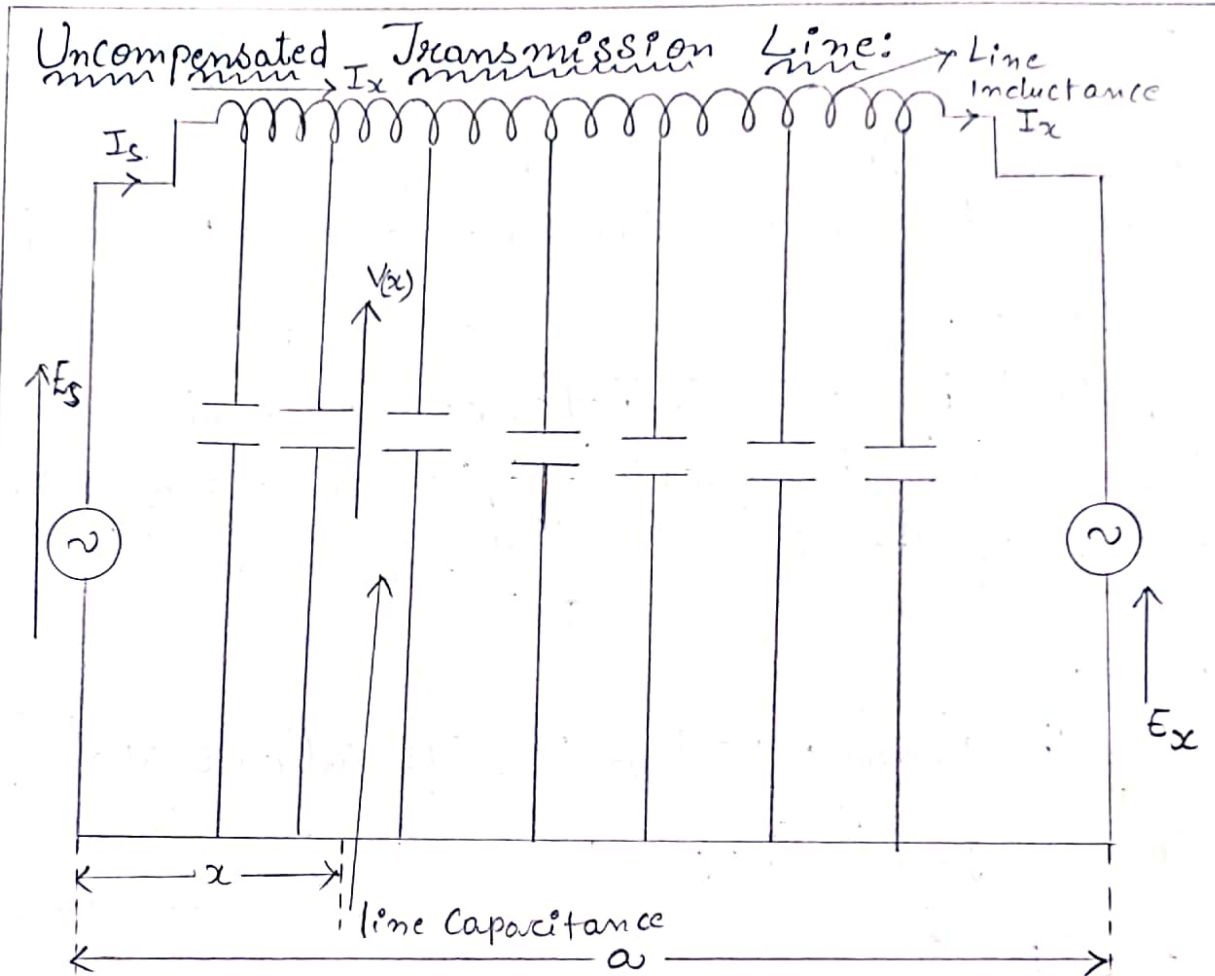
$$\frac{1}{2\pi \times 50 \times 10.26} = C$$

$$C = 3.10 \times 10^{-4} \text{ F}$$

$$= 310 \mu\text{F}$$

Capacitance of each unit

$$= \frac{310}{5} = 62 \mu\text{F}$$



The transmission line is having no compensation is called Uncompensated.

A transmission line is characterised by 4 distributed parameters.

- 1) Series resistance
- 2) Shunt Conductance
- 3) Line Inductance
- 4) Line Capacitance

All the four parameters representing the function of Conductor type, Conductor Spacing, Conductor Size, Conductor height, Conductor temperature & pressure.

The behaviour of transmission line is explained by series inductance and shunt capacitance. The equation representing propagating energy of the line is  $\frac{d^2 V}{dx^2} = \pi^2 V$

where

$V$  = the phasor voltage and its value is  $\hat{V} e^{\frac{j\omega t}{\sqrt{2}}}$

$$\pi^2 = (\pi + j\omega L)(G + j\omega C)$$

$$\pi = j\beta$$

where  $\beta$  is the electrical strength of line expressed in radians or wavelength

$$\beta = \omega\sqrt{LC} = \frac{2\pi f}{u}$$

where  $u$  = Velocity of light =  $3 \times 10^8$  m/sec

The solution of voltage and current at point 'x'.

$$V(x) = V_n \cos \beta (\omega - x) + j I_n Z_0 \sin \beta (\omega - x)$$

$$I(x) = j \frac{V_n}{Z_0} \sin \beta (\omega - x) + I_n \cos \beta (\omega - x)$$

Surge Impedance or Characteristic Impedance  $Z_0 = \sqrt{L/C}$   
For overhead lines Surge Impedance =  $400 \Omega$



Surge Impedance  $Z_0$  (or)  $Z_c$ :

Surge Impedance is the apparent impedance of an infinitely long line or the ratio of voltage & current at any point of the line.

$$Z_0 = \frac{V_H}{I_H}$$

$$V_H = I_H Z_0$$

$$Z(x) = \frac{V(x)}{I(x)}$$

$$Z(x) = \frac{V_H \cos \beta(\omega - x) + j I_H Z_0 \sin \beta(\omega - x)}{j \frac{V_H}{Z_0} \sin \beta(\omega - x) + I_H \cos \beta(\omega - x)}$$

Substitute  $V_H = I_H Z_0$  in the above equation

$$\begin{aligned} Z(x) &= \frac{I_H Z_0 \cos \beta(\omega - x) + j I_H Z_0 \sin \beta(\omega - x)}{j \frac{I_H Z_0}{Z_0} \sin \beta(\omega - x) + I_H \cos \beta(\omega - x)} \\ &= \frac{Z_0 [I_H Z_0 [\cos \beta(\omega - x) + j \sin \beta(\omega - x)]]}{I_H Z_0 [j \sin \beta(\omega - x) + \cos \beta(\omega - x)]} \end{aligned}$$

$$Z(x) = Z_0$$

From the above equation it is seen that a flat voltage profile is maintained and voltage & current are in phase with each other.

Natural load or Surge Impedance loading (SIL) is

$$P_0 = \frac{V_0^2}{Z_0}$$

\*  $V_0$  indicates line to neutral voltage.

\* SIL varies square of the voltage.

\* The need of SIL is to maintain a flat voltage profile and uniform stress at any point on the line.

Voltage & Current profile in open circuit condition

For open circuit  $I_R = 0$

$$V(x) = V_R \cos \beta(\omega - x)$$

$$I(x) = j \frac{V_R}{Z_0} \sin \beta(\omega - x)$$

Receiving end voltage  $E_S = V_R \cos 0$

$$V_R = \frac{E_S}{\cos 0}$$

Receiving end current  $I_S = j \frac{V_R}{Z_0} \sin 0$

$$= j \frac{E_S}{\cos 0 Z_0} \sin 0$$

$$= j \frac{E_S}{Z_0} \tan 0$$

$$0 = \beta(\omega - x)$$

Receiving end voltage  $V(x) = \frac{E_S}{\cos 0} \cos \beta(\omega - x)$

$$V(x) = E_s$$

$$I(x) = j \frac{E_s}{Z_0} \tan \beta (\omega - x)$$

$$V_s = V_r$$

$V_0$  Power is get transferred

- \* The requirement of reactive power decides the rating of compensating equipment.
- \* If inductive load is connected at the sending end synchronous machine will absorb line charging reactive power.
- \* During Uncompensation synchronous machine will absorb or generate the difference between line & local load reactive power.

### Compensation of Lines:

Compensation means use of electrical circuits for modifying electrical characteristics of the line.

#### Objectives:

- \* Ferranti effect is avoided so that a flat voltage profile is maintained for all the loading conditions.
- \* Increasing power transfer capability so that stability is obtained.
- \* Under excitation of alternators is avoided so that a proper management of reactive power is done.
- \* The compensation is taken in terms of length of the line & power to be transmitted.



### Maintaining Flat Voltage Profile:

- \* Flat voltage profile on the line can be achieved if the loading of the line corresponds to surge impedance loading.
- \* To achieve a flat voltage profile the compensating device should be chosen so that virtual surge impedance ( $Z_c$ ) of the line should give virtual natural loading equal to actual loading. But actual load varies w.r.t. to time. So compensating device should also vary so that effective impedance matches with actual loading i.e;

$$P_c = \frac{V_H^2}{Z_c} \Rightarrow \text{actual load } V_L$$

- \* Compensating devices are suitable collection of capacitors & inductors in the line. The compensation carried out with varying surge impedance compensation.

### Increasing Power transfer Capability:

It can be achieved by inserting a series capacitor at suitable location. So that line inductive reactance is reduced which is equivalent to reduction in effective length of the line it is called line length compensation.

Under Excited of Alternators is avoided:

It can be achieved by dividing the transmission line into shorter sections known as Compensation by section listing. It can be achieved by inserting Voltage Compensators at intervals along the line.

Same power transmitted is same to all the sections the maximum power will be decided by smallest section and thereby increases the power transfer capability hence stability limit is increased.

Compensators are classified into two types

- 1) Active Compensator
- 2) Passive Compensator

Active Compensators are Thyristor Control Reactor (TCR), Thyristor Switched Capacitors (TSC), Synchronous capacitor.

Passive Compensators are Shunt Capacitors, Series Capacitors & Shunt reactors. Compensators are used in discrete location & their effect is uniformly distributed along the line.

Shunt Compensation:

$$\text{Swing Impedance } Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{X_L \times X_C}$$

Let  $L_{sh}$  be the Shunt inductance used as compensator.  
Then Net reactance is



$$\begin{aligned}
 j\omega C' &= j\omega C + \frac{1}{j\omega L_{sh}} \\
 &= j\omega C - \frac{j}{\omega L_{sh}} \\
 &= j\omega C - \frac{j}{\omega L_{sh}} \times \frac{j\omega C}{j\omega C} \\
 &= j\omega C \left[ 1 - \frac{j}{\omega L_{sh} j\omega C} \right] \\
 &= j\omega C \left[ 1 - \frac{1}{\omega^2 C L_{sh}} \right]
 \end{aligned}$$

$$j\omega C' = j\omega C [1 - \gamma_{sh}]$$

$\gamma_{sh}$  = degree of shunt compensation

$$\gamma_{sh} = \frac{1}{\omega^2 C L_{sh}}$$

Modified Surge impedance

$$Z_c' = \sqrt{\frac{j\omega L}{j\omega C'}} = \sqrt{\frac{j\omega L}{j\omega C [1 - \gamma_{sh}]}} = Z_c \frac{1}{\sqrt{1 - \gamma_{sh}}}$$

$$Z_c' = \frac{Z_c}{\sqrt{1 - \gamma_{sh}}}$$

If shunt capacitance is added  $\gamma_{sh}$  will be negative and shunt inductance increases virtual surge impedance and reduces virtual surge impedance loading, shunt capacitance reduces virtual surge impedance.



### Series Compensation:

Let  $C_{se}$  be the Series Compensator.

Then net reactance

$$j\omega L' = j\omega L + \frac{1}{j\omega C_{se}}$$

$$= j\omega L - \frac{j}{\omega C_{se}}$$

$$= j\omega L - \frac{j}{\omega C_{se}} \times \frac{j\omega L}{j\omega L}$$

$$= j\omega L \left[ 1 - \frac{1}{\omega^2 L C_{se}} \right]$$

$$= j\omega L [1 - \gamma_{se}]$$

$\gamma_{se}$  = degree of Series Compensation

$$\gamma_{se} = \frac{1}{\omega^2 L C_{se}}$$

Virtual Surge Impedance

$$Z_c' = \sqrt{\frac{L}{C}} = \sqrt{\frac{j\omega L'}{j\omega C}}$$

$$Z_c' = \sqrt{\frac{j\omega L(1 - \gamma_{se})}{j\omega C}}$$

$$\boxed{Z_c' = Z_c \sqrt{1 - \gamma_{se}}}$$

Considering both Series & Shunt Compensation.

$$\text{Virtual Surge Impedance } Z_c' = Z_c \sqrt{\frac{1 - \gamma_{se}}{1 - \gamma_{sh}}}$$

Virtual Surge Impedance loading

$$P_c' = P_c \sqrt{\frac{1 - Y_{sh}}{1 - Y_{se}}}$$

$$\text{Wave number } \beta = \sqrt{(1 - Y_{se})(1 - Y_{sh})}$$

Inductive Shunt Compensation increases Virtual Surge Impedance and Capacitive Compensation decreases Virtual Surge Impedance.

If Inductive Compensation is 100%. Virtual Surge Impedance becomes infinity for zero loading conditions and maintains a flat voltage profile.

During loading conditions Shunt capacitor is installed for maintaining a flat voltage profile.

Transmission Line

8) + Compensator:

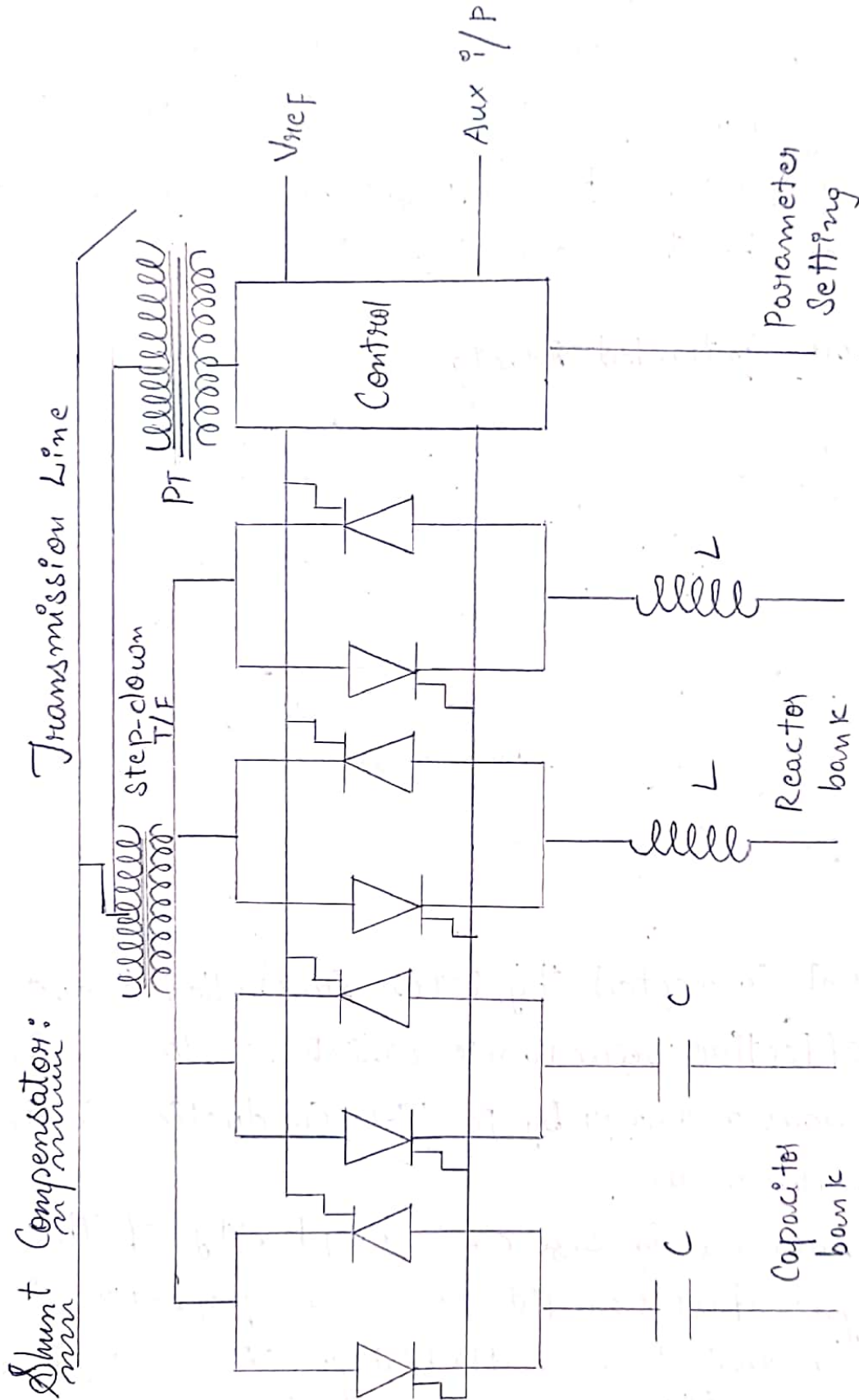
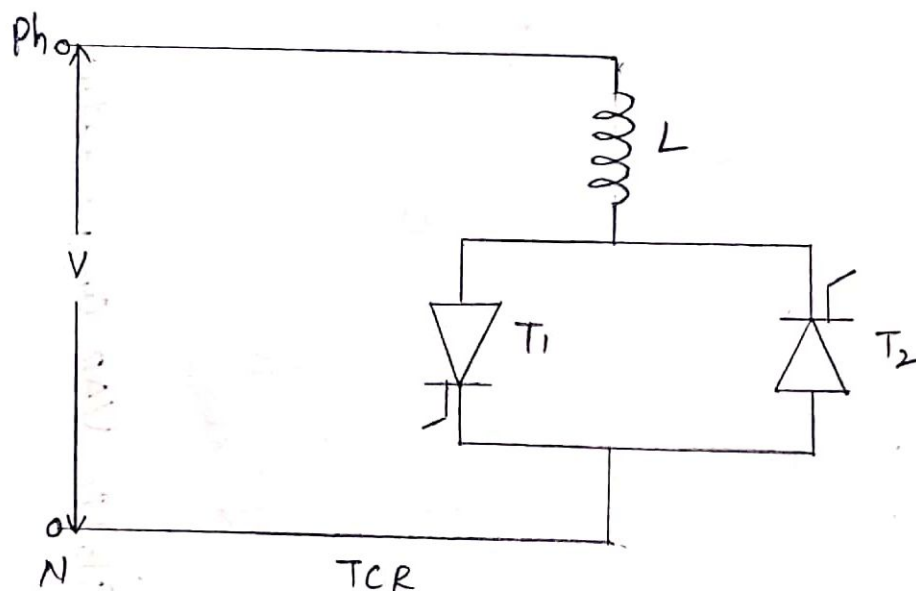


Fig: Shunt Connected Static VAR Compensator.



A Shunt Connected Static VAR Compensator consists of Thyristor Controlled reactor and Thyristor Switched Capacitor. With proper Coordination of Capacitor Switching, reactive control VAR o/p can be varied continuously b/w Capacitive and Inductive rating of the equipment. The compensator mainly controls voltage of the transmission line.

### Thyristor Controlled Reactor



A Shunt Connected thyristor controlled inductor has an effective reactance which is varied in a continuous manner by partial conduction control of thyristor wall.

It increase in size and complexity of the power system fast reactive power compensation is needed for maintaining stability. TCR is very fast it takes few milliseconds for producing response.

Reactive power Compensation using TCR becomes popular and used as Static Compensator.

A Fixed reactor having inductance ' $L$ ' and a bidirectional thyristor valve. The thyristor valve can be turned on by applying gate signal and automatically blocked after the AC current crosses zero. The current in the reactor gets varied by changing the firing angle. Partial conduction is obtained with a higher firing angle and reduces the fundamental component of current. This is equivalent to increase in inductance of the reactor.

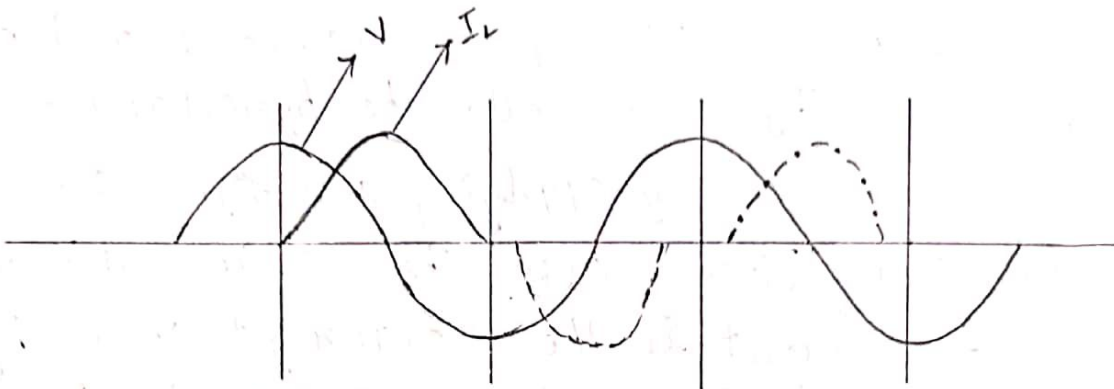
The current in the reactor is purely inductive and lags the supply voltage by approximately  $90^\circ$ . TCR generates harmonics (odd harmonics) and can be filtered out by means of filters.

#### TCR Characteristics

- 1) Generates harmonics
- 2) No transients
- 3) Continuous Control.

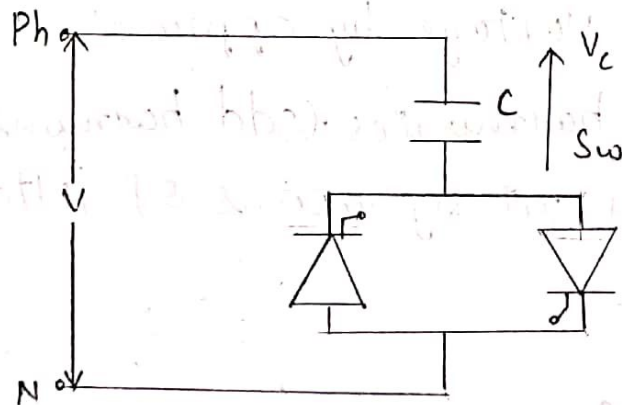


## TCR waveform:

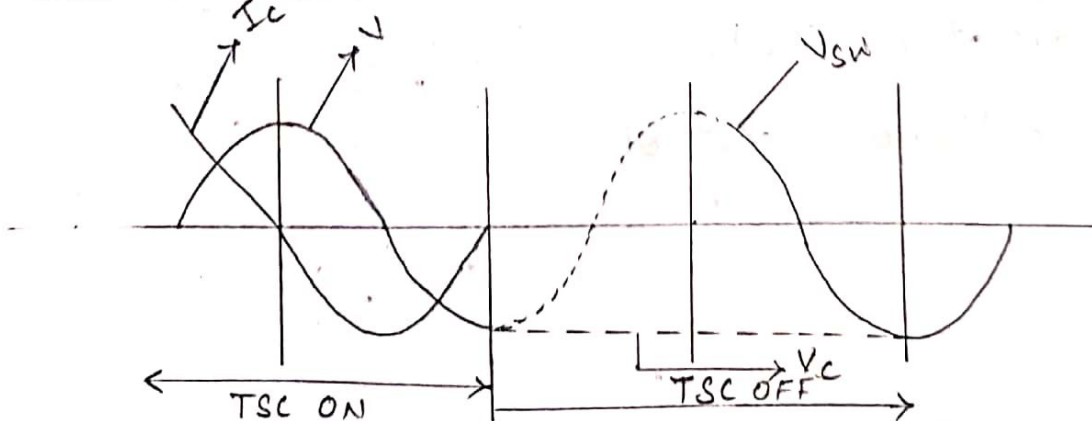


————— Voltage  
 - - - - -  $I_t$  for  $\alpha_1 \alpha = 90^\circ$   
 - . - . - .  $I_t$  for  $\alpha_2$   
 —————  $I_t$  for  $\alpha$

## Thyristor Switched Capacitor



## TSC waveform





A Shunt Connected Thyristor Switched Capacitance has a effective reactance and can be varied in a stepwise manner. By full or zero conduction of thyristor valve TSC is a static compensator that has thyristor based switches are used to switch in ON or OFF the capacitor unit depending upon KVAR requirement.

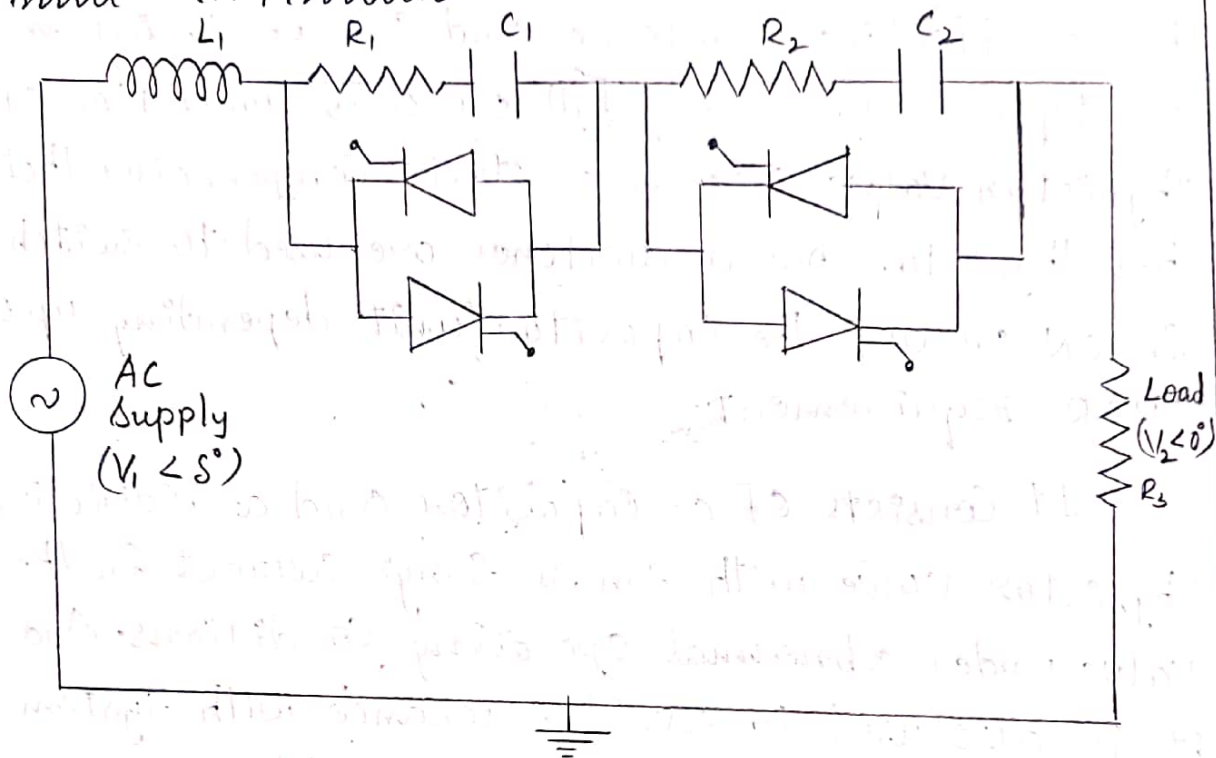
It consists of a capacitor and a bidirectional thyristor valve with small surge current in the valve under abnormal operating conditions. And it is also used to avoid resonance with system impedance at particular frequency. The problem at transient is avoided by keeping the capacitors charged to +ve peak or -ve peak voltage when they are in stand by state.

The switching ON transient is selected when the same polarity exists in the capacitor voltage and ensures that natural zero passage of the capacitor current & switching OFF the capacitor is done by reducing the firing angle.

Characteristics:

- \* No transients
- \* No harmonics
- \* Stepped control
- \* Low losses
- \* Redundancy & Flexibility

## Series Compensation:



For a fixed degree of series compensation in the TCR scheme the current in the reactor gets increased and TCR is designed with maximum admittance.

In TSC scheme increasing the number of capacitor units controls degree of series compensation and capacitor bank is controlled by thyristor valve. The thyristor valve becomes ON when the AC voltage crosses zero and it becomes OFF when zero current passes in the capacitor.

Initially the capacitor is charged in some voltage for reducing transients and to protect scr a resistor is connected in series with capacitor.



## Characteristics

- Minimizing the losses
- Reducing the loop flow
- Elimination of over loads.
- Optimal load sharing b/w parallel ckts

## Needs OF FACTS Control:

FACTS: Flexible AC Transmission System

- \* For increasing performance & utilization of existing transmission system FACTS devices are used.

## Functions:

- \* Increases Transient Stability
- \* Powerflow Control
- \* Voltage Regulation.

FACTS means collection of controllers

## Types:

### i) Series Controller:

It is a variable impedance which inject voltage in the transmission system (Series).

### ii) Shunt Controller:

It is a variable impedance source inject current into the transmission system.